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ABSTRACT

By appointment of the Conference Board of the Mathematical Sciences, a committee was established to investigate the status of mathematical education at the pre-college levels. Drawing on the data gathered and published by the National Assessment of Educational Progress, the National Center for Educational Statistics' survey of course offerings and enrollments at public secondary schools, and the American Institutes for Research's survey concerning computing activities in secondary education, as well as the committee's own studies of statewide objective and testing programs and of elementary school curricula and instruction, the report addresses a broad spectrum of issues. Chapters of the report are devoted to a summary and analysis of the curricular reforms during 1955-1975, identification and discussion of current curricular emphases, analysis of alternative instructional situations and patterns, discussion of teacher education, and statement of recommendations for policy changes as well as for research and development. Two overriding policy recommendations advocate the avoidance of dichotomizing mathematics (e.g., "old" vs. "new") and the commitment to quality mathematical education for all. Suggestions concerning curriculum, teacher education, evaluation, and research are presented in detail. (SD)

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PREFACE

In American schools today nearly everyone of the roughly 30 million students in grades K-8 studies basic mathematics. The instruction is provided by at least 750,000 different teachers with extremely varied preparation, experience, and interest in mathematics teaching. At the high school level, about 9 million students are enrolled in hundreds of different special courses ranging from remedial arithmetic and elementary algebra to computer science and calculus. These students are taught by at least 75,000 different teachers -- most specialists in mathematics, but again with widely varying abilities, objectives, and methods for mathematics instruction.

Despite this immense diversity of student, teacher, and institutional goals and capabilities, the continuing debate that influences program priorities and resource allocation makes frequent reference to supposed "national trends" in the practices and attainments of mathematics teaching. Because many of its member organizations have a vital interest in the health of school mathematics, the Conference Board of the Mathematical Sciences appointed in May, 1974 a National Advisory Committee on Mathematical Education (NACOME). The Committee was directed to prepare an overview and analysis of U.S. school-level mathematical education -- its objectives, current practices, and attainments.

The members of NACOME include Robert P. Dilworth, James F. Gray, Shirley Hill (Chairperson), John L. Kelley, Peggy Neal, Jack Price, and Rheta Rubenstein. This Committee brought a wide range of experience and expertise to the very difficult task of surveying and analyzing practices and issues in mathematics teaching from kindergarten through senior high school. Taken together, their current activities span classroom teaching of mathematics from elementary grades through graduate school, mathematical research, pre- and in-service teacher education, curriculum development and implementation projects, supervision of curriculum and instruction, and evaluation of programs in school mathematics and teacher education. Their experiences include such activities in rural, suburban, and urban schools from across the United States and several foreign countries. Furthermore, they have been active as leaders of state and national professional organizations in mathematics and mathematics teaching.

NACOME invited input for the study from all concerned professional groups and individuals and has profited immensely from information and insight they have provided. We are particularly grateful to the officers, committees, and administrative staff of the National Council of Teachers of Mathematics for assistance throughout the study, including financial support for an exploratory survey of elementary school mathematics teaching practice that was vital to our work. The Association of State Supervisors of Mathematics provided important assistance in our effort to assemble an overview of emerging programs of objectives and assessment at the state level. The documents and personal observations provided by these state leaders were of immeasurable value in giving national perspective to current practices and issues in mathematics teaching.

The NACOME overview has also drawn on information from a number of recently published reports, including the survey of Course Offerings and Enrollments in Public Secondary Schools of the National Center for Educational Statistics, a survey of Computing Activities in

Secondary Education by the American Institutes for Research, and various technical reports from the 1972-73 National Assessment of Educational Progress in mathematics. We are in debt to all these organizations for access to advance copies of these important documents.

In portraying current trends in curriculum and instruction we have utilized information provided generously by directors of many major research and development projects mentioned in Chapters 2 and 3 of our report. Special insight into the progress and promise of computers in mathematics education came from members of the 1972 CBMS Committee on Computer Education and several other consultants. Similar advice on the statistics and probability curriculum came from the Joint Committee of the American Statistical Association and the National Council of Teachers of Mathematics. The NCTM Commission on the Education of Teachers of Mathematics provided information from an exploratory survey of teacher education activities and, through deliberations at their meetings, helped us to identify important issues in teacher education. In attempting to assemble an accurate picture of trends in the practice and results of standardized testing we were aided by information from developers of several major tests. Publishers of widely used mathematics textbooks provided similar information on the content and style of instructional materials. Research on many broad questions was expedited considerably by assistance of the ERIC center at Ohio State University.

Overall support for the NACOME project was provided by the National Science Foundation. We are particularly indebted to Professor James Wilson of the University of Georgia for his continual input of information and insight throughout the study -- initially as a member of NACOME and then as NSF liaison for the project during 1974-75 when he was on leave at the Foundation.

We are deeply indebted in myriad ways to our Project Director, Truman Botts, and our Executive Secretary, James Fey. Truman Botts' efficient organization and administration of the project have made the work of the committee not only possible but enjoyable. And it is impossible to overstate the crucial nature of the role James Fey played in this study and the impressive manner in which he played it. The collection and organization of a massive amount of information was a monumental task and whatever positive contribution this report may make can be attributed largely to his judgment and efficiency.

Shirley Hill
Chairperson, NACOME

November 1975

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INTRODUCTION

For nearly twenty years mathematics has held a position of unprecedented favor and prominence in school curricula. Spurred by technological competition with the Soviet Union, federal agencies and private foundations have invested heavily in mathematics curriculum development in grades K-12. Mathematicians, teacher educators, and classroom teachers have collaborated on projects whose acronyms have been standard vocabulary in education -- SMSG, UICSM, UMMaP, Madison Project, SSMCIS. The goal has been major reconstruction of the scope, sequence, and pedagogy of school mathematics. To facilitate implementation of the new programs the National Science Foundation joined state and local educational institutions in a massive effort to up-date mathematical competence of teachers. Yet today mathematics teaching is a troubled profession. Strident criticism from colleagues and the popular press has forced many mathematics educators to re-examine the wisdom and effectiveness of their varied innovations known collectively and popularly as "new math".

Critics of these curricula complain that reform has produced programs that are excessively formal, deductively structured, and theoretical -- ignoring essentially intuitive interaction of mathematics with its applications. They argue that "new math" curricula fail to meet the needs for basic mathematical literacy of average and low ability students. These challenges have led to recent proposals for interdisciplinary and career oriented curricula in which mathematical topics enter where proven valuable for scientific, technological, or consumer problem solving. Recent emergence of low cost electronic calculators has added another dimension of uncertainty to curricular decision making, challenging the traditional priority

assigned to arithmetic skill development in grades K-8. At a higher level, growing accessibility of computers raises serious questions about the organization and emphasis of topics in algebra, geometry, and calculus.

Challenges to recent and traditional curriculum priorities are paralleled by an equally spirited debate over alternative instructional styles. The model for "new math" teaching was Socratic dialogue guiding students to discovery of key concepts. There is widespread doubt that this teaching style ever gained much classroom acceptance, and mathematics teachers today are struggling to find proper uses for a wide range of competing methods -- including laboratory, small group, individualized, and computer mediated instruction. As the management concepts of performance objectives and accountability gain influence on educational practice, mathematics teachers face the task of specifying and measuring accurately the full range of their expected instructional effects. Teacher educators face a similar challenge in the growing movement toward performance-based teacher education.

The most public controversy in mathematics education centers on conflicting reports of student achievement in current mathematics programs. Critics cite evidence of declining mathematics scores in California, New Hampshire, and New York and on the nationally known College Board tests. Defenders of "new math" offer contrary evidence, point to declining scores in other subject areas, and argue that measurement of mathematical abilities is an extremely complex task, interwoven with many crucial factors of school situations. Yet there is an uneasiness, widespread among teachers and parents, that students are not up to par in mathematics. The concern is compounded by disagreement and confusion over proper goals for school mathematics.

Recent changes in American mathematics education have been part of a worldwide movement to up-date the content and teaching of school

mathematics. The main themes of United States reform have counterparts in similar, often more daring, developments in Canada, Japan, the Soviet Union, Germany, Denmark, Belgium, France, and Great Britain. The "new math" is now subject to professional and public criticism in many of these countries too. The process and product of curriculum change in most foreign countries is easy to describe. Central ministries of education direct curriculum writing teams that produce syllabi and text materials for use throughout the country. The ministries control teacher certification and in-service education, and they commonly give comprehensive national examinations measuring achievement in the syllabi. In such situations discussion of mathematics programs can focus on reasonably well defined curricula and pedagogy for which systematic effectiveness measures are available.

In contrast, for American mathematics education fundamental decisions on curriculum content and instructional practice are traditionally made at the local school level -- usually allowing classroom teachers substantial freedom in choice of materials and methods. The United States does not have official national mathematics syllabi, teacher certification standards, or achievement tests. School mathematics programs evolve in response to a vast array of professional advisory group and public pressures -- operating on a patchwork information base.

Since its founding in 1920, the National Council of Teachers of Mathematics (NCTM) has been the forum for discussion of issues in school mathematics. Its commissions have produced periodic influential guidelines for curricular change and teacher education. The Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America (MAA) has published guidelines that have had some impact on teacher education. Test syllabi of the College Entrance Examination Board have long exerted clear (if unofficial) influence on the character of college preparatory mathematics programs, and widespread use of a few commercial standardized

achievement batteries almost certainly plays a similar role at lower grade levels. New curricular ideas have commonly made their way into classroom use through the highly competitive commercial textbook marketplace where authors and publishers search for a winning blend of traditional material and fashionable innovation. The strategy of federal support for curriculum development, a phenomenon of the 1955-1975 period, has been to generate alternative innovative programs, not a single approved model.

Given the diverse character of United States school mathematics programs and the incomplete data base for assessing nature and quality of the programs, how can teachers, supervisors, teacher educators, or the concerned public make reasoned judgments of competing curricula and instructional practice? In June 1974, at the request of several constituent organizations, the Conference Board of the Mathematical Sciences appointed a National Advisory Committee on Mathematics Education (NACOME). With financial support from the National Science Foundation, NACOME has attempted to assemble a comprehensive overview and analysis of the current status of mathematics education K-12 -- its objectives, current and innovative practices, and attainments. In light of this overview and analysis, the Committee has formulated specific recommendations on the needs and opportunities for improvement in school-level mathematical education.

The Committee's work was timed to capitalize on publication of three extremely helpful statistical studies. The National Assessment of Educational Progress (NAEP) conducted its first mathematics assessment in 1972-73. Data analysis was completed and technical reports published in 1975 -- giving the first national profile of mathematical abilities possessed by 9, 13, and 17 year olds. The results of this assessment and the mathematics objective framework planned for 1977-78 national assessment have important implications for current debate on the goals and attainments of school mathematics.

Also in 1975, the National Center for Educational Statistics (NCES) published results from its 1972-73 survey of course offerings and enrollments in public secondary schools. The most recent previous report of this census-like school practice survey was in 1960, at the very beginning of an innovative decade in school programs. Together with patterns in earlier surveys (back to 1890) these data give valuable perspective to current offerings and trends.

September 1975 American Institutes for Research (AIR) published a survey of Computing Activities in Secondary Education. Along with an earlier (1970) AIR study, these data give valuable insight into the impact that computing is making and the accessibility of future computer based innovations.

NACOME has examined carefully a fourth source of national perspective on school mathematics, the growing trend toward state level objective and testing programs. With support of the Association of State Supervisors of Mathematics (ASSM) we were able to survey and analyze the current goals for mathematics instruction expressed in state syllabi and to synthesize varied data on achievement arising from state assessment efforts.

These studies gave insight into the goals and attainments of school mathematics. But very few data exist to help characterize the fundamental part of any program, the actual patterns of classroom instructional activity K-12. Here the NCTM helped NACOME with financial support for an exploratory survey of curricula, teaching style, and teacher preparation in elementary school mathematics. Data from this survey helped the Committee construct more reliable descriptions or mathematics teaching today, and they suggest important directions for future research.

While many educators and laymen criticize the priorities and achievements of current school mathematics programs, many others

applaud recent innovations and find little cause for alarm in achievement reports. The issues are complex and the pertinent information must be drawn from widely disparate sources. In appraising each of the data sources and formulating recommendations for progress in school mathematics, the Committee has benefited from contributions of countless individuals and professional groups concerned about the future of mathematics teaching. Our goal has been to clarify issues of curriculum and teaching, to identify the range of viable alternative practices, and to survey and evaluate information useful in choosing among the alternatives. Several of the issues have a long history in mathematics education. Serious gaps in the available information prevent definitive resolution of many questions. However, we have striven for a carefully reasoned synthesis of current practices, proposals, and evidence of achievement that we hope will provide valuable national perspective for crucial decisions facing mathematics education today.

CHAPTER 1. MATHEMATICS CURRICULUM REFORM 1955-1975

Beginning in the mid 1950's, efforts to improve school mathematics emphasized the content of instruction: new mathematical topics, new organization and grade placements of traditional topics. Public and professional debate over school mathematics has focused on the merits of these curricular innovations. To clarify issues and alternatives in this debate, the present survey and analysis sought answers to the following questions:

> What have been the goals and rationale for recent curriculum developments?
>
> How accurately and broadly are these goals realized in current school programs?
>
> What potent forces and promising innovations should shape mathematics curricula in the years immediately ahead?

1.1 Goals of the Reform

The initial "new math" curriculum development projects were directed at the high school program for college preparatory students. Rationale and design for the major innovations were formulated at a series of American and international conferences in the late '50s and early '60s. The most carefully reasoned and influential report came from the Commission on Mathematics of the College Entrance Examination Board (CEEB). The Commission Report, published in 1959, made the case for reform with the following arguments:

1. Extensive development of new concepts and methods has dramatically altered the structure of pure mathematics.

2. The spectacular growth of pure mathematics has been accompanied by successful application of both classical and recently discovered ideas to previously non-mathematized disciplines like biological, social, and management sciences.

3. To meet the fast growing need for mathematically sophisticated scientific manpower, secondary school instruction must present new content organized in a way that helps bring students more efficiently to the frontiers of pure and applied mathematics. [1]

The Commission Report suggested that topics from logic, modern algebra, probability, and statistics be among the new content of school instruction. But the major recommendations focused on efficient ways to reorganize the treatment of traditional school topics. Plane and solid geometry were to be integrated in a single course; trigonometry was to merge with the conventional second year algebra course; and the entire curriculum was to be unified through judicious use of deductive method, the process of pattern searching, and structural concepts like set, relation, and function. Through curriculum development efforts of the School Mathematics Study Group (SMSG) and the University of Illinois Committee on School Mathematics (UICSM) the main themes of the Commission Report were soon transformed into model classroom course materials.

As the Commission Report was beginning to influence curricula for college preparatory high school students, the 1963 Cambridge Conference endorsed the main lines of school mathematics reform in far more daring Goals for School Mathematics, grades K-12. The Cambridge Report claimed to express only tentative views on the nature of a good school curriculum for the future. However, its striking proposals to accelerate and enrich traditional curricula have undoubtedly influenced all subsequent curriculum research and development, particularly at the elementary level where innovation had only begun in projects at Stanford University and Cleveland, Ohio schools.

The CEEB and Cambridge proposals were generated largely in response to developments within mathematics. But justification on psychological grounds came with the 1960 publication of Jerome Bruner's The Process of Education. This eloquent analysis of psychological issues in science instruction was persuasive justification for teachers to emphasize conceptual understanding of mathematical methods -- understanding to be conveyed by stress on unifying structures of the discipline.

If the high school mathematics program was to incorporate new content in a curriculum organized around powerful but abstract structuring concepts and processes, preparation in elementary and junior high school had to change too. At the same time Bruner forecast improved acquisition and transfer through focus on the structure of disciplines, he re-emphasized concern for psychological issues like readiness, intuitive versus analytical thinking and concrete versus formal experience in learning. These curricular and psychological forces had a strong impact on thinking about goals for mathematics instruction K-8.

In materials developed by the University of Maryland Mathematics Project (UMMaP) and SMSG, the traditional junior high review of practical arithmetic* was augmented by informal geometry, probability, algebra, and analysis of number and numeration structures.

SMSG introduced similar topics in its K-6 experimental curricula -- probing Bruner's hypothesis that "any subject can be taught effectively in some intellectually honest form to any child at any stage of development." [2] Influenced by Piaget's theories of cognitive development, curriculum developers began careful analysis of what could be learned by children at various stages of development. These

---

* held over from days when the junior high school was for many students their last formal education.

studies were reflected in new designs for elementary mathematics programs in which the preferred "intellectually honest form" of instruction was often some concrete model of mathematical ideas.

Cardinal number concepts were developed by variations on the Piagetian one-to-one correspondence tasks; ordinal number concepts were developed by manipulation in various systems of rods or blocks. The clear thrust was to replace an elementary curriculum emphasizing rote learning of arithmetic by psychologically appropriate instruction.

1.2 Implementation of New Programs

Critical appraisal of curricular changes is a complex task involving interdependent judgments of what should and can be proper goals of instruction. Recent public debate over the merits of "new math" has suffered from the participants' tendency to caricature both broad themes and specific content of new K-12 programs. Furthermore, in judging the effectiveness of new programs, critics tend to assume that the curricular designs of major conferences and experimental projects have become uniform classroom practice throughout the country. As Bruner points out, when major curriculum experimentation got underway in 1957-1960 developers faced a twofold problem: "first, how to have the basic subjects rewritten and their teaching materials revamped in such a way that the pervading powerful ideas and attitudes relating them are given a central role; second, how to match the levels of these materials to the capacities of students of different abilities at different grades in school." [3] Their efforts took many different forms, and as the initial material filtered into commercial texts and widespread classroom use the variation increased dramatically.

Choosing the year 1972-73 (date of the NCES survey of course offerings and the National Assessment) as a benchmark, NACOME has tried to assemble an estimate of the national impact that "new math" goals have had on actual school curricula.

Senior High School. Carrying influential endorsement of the College Entrance Examination Board, recommendations of the Commission on Mathematics were immediately realized in high school programs. The widely used SMSG texts stressed the themes of

--treating inequalities along with equations
--structure and proof in algebra
--integrated plane and solid geometry with coordinate methods
--integrated algebra and trigonometry
--a twelfth grade course in elementary functions.

As a result, one 1965 survey of College Board examinees [4] showed that, with the exception of probability and statistics, content recommendations of the Commission had entered the programs of many schools. Impact of the Commission plan to prepare students for collegiate study at the level of calculus can also be inferred from the decline in college offerings of pre-calculus courses evident prior to 1970, a trend which shows some signs of reversal today.

The College Board program and survey indicate changes for a targeted but narrow sample of secondary mathematics students. Changes for less able students in conservative systems were undoubtedly slower and less pronounced. However, a "before and after" contrast of leading commercial texts reveals clearly the broader impact of Commission recommendations by 1972-73. And data from NCES surveys of course offerings and enrollments [5] confirm the trend toward new course organization.

The 1960 survey showed a growing proportion of secondary students enrolled in mathematics courses. In 1949 only 65% of secondary students (7-12) were enrolled in some mathematics course; by 1960 the figure had risen to 73% . Individual increases were prominent in advanced general mathematics, plane geometry, advanced algebra, and trigonometry -- indicating that students were already beginning to seek more extensive preparation for college level science study. Furthermore, the 1960 survey revealed that 2.3% of all twelfth graders were enrolled in advanced mathematics courses such

as calculus (3,723 students in 170 schools), probability and statistics (1,605 students in 58 schools), mathematical analysis (10,430 students in 347 schools), college mathematics (3,260 students in 108 schools), and analytic geometry (4,153 students in 203 schools).

Though data from the 1972-73 survey have been reported somewhat differently, making comparisons difficult, they reveal some very interesting patterns. The number of students taking a second course in algebra or the new integrated algebra/trigonometry course had risen to nearly equal the number of students taking elementary algebra (about 2 million students at each level). The algebra/trigonometry format captured 40% of the advanced algebra registrations. Over 260,000 students were in calculus or other advanced level mathematics courses (four times the 1960 figure). Some 500,000 students were described as studying one of the various experimental curricula (SMSG, SSMCIS, UICSM, etc.). Furthermore, the variety of mathematics courses available to prospective students had expanded dramatically since 1960. (see table 1)

The impact of Commission recommendations on thinking about proper curricula for schools is evident in the decline of solid geometry offerings (coupled with rise of unified plane and solid geometry courses), growth of the advanced algebra/trigonometry option, and appearance of many different twelfth year options in advanced mathematics. These offering and enrollment data are paralleled by patterns of change in state and local curriculum guides and mathematics objectives, most of them formulated during the period of curricular evolution.

Table 1

U. S. Public Secondary Schools Offering Various Mathematics Courses, Given as a Percent of Total Number of Schools. [6]

	School Enrollment Less Than 500		School Enrollment Between 500 and		School Enrollment Greater Than 1000	
	1960	1972	1960	1972	1960	1972
Math 7 General	54%	51%	60%	57%	45%	34%
Math 8 General	57%	54%	64%	61%	49%	36%
H.S. General I	62%	47%	73%	55%	70%	62%
H.S. General II	24%	26%	33%	30%	46%	43%
Pre-Algebra	-	.4%	-	2%	-	4.5%
Elementary Algebra	84%	62%	89%	72%	93%	83%
Intermediate or Advanced Algebra	63%	53%	68%	54%	72%	68%
Advanced Algebra/ Trigonometry	.6%	9%	1.8%	12%	4%	24%
Plane Geometry	68%	43%	67%	36%	72%	48%
Plane/Solid Geometry	-	20%	-	21%	-	34%
Solid Geometry	22%	3%	39%	3%	47%	4%
Trigonometry	34%	20%	52%	17%	59%	30%
Remedial Math 7-8	-	8%	-	13%	-	8%
Remedial Math 9-12	-	13%	-	19%	-	28%
Consumer Math	-	2%	-	3%	-	4%
Applied Math	-	3%	-	6%	-	13%
Computer Math	-	1%	-	4%	-	20%
Advanced Math	2.5%	6%	9%	8%	18%	14%
Math Analysis	*	8%	*	12%	*	23%
Elementary Functions	-	1%	-	2%	-	6%
Probability & Stat.	*	1%	*	3%	*	10%
Calculus	*	4%	*	9%	*	27%
Number of Schools	15172	8696	4364	7129	3297	6882

* These and other courses were offered in 1960 (see pages 6-7), but data were not given in the 1960 report in a way that could be fit to the comparison made here.

\- These course titles did not appear in the 1960 survey report.

Despite this formal evidence of "new math" adoption in schools, appraisal of curricular effectiveness is stymied by a deeper question on which helpful pertinent data are sparse -- were the content and organizational innovations realized in the classroom presentations and evaluations of any large fraction of mathematics teachers?

> Do teachers really emphasize the structural properties of number systems that underlie algebraic techniques, or do they continue to stress rote memorization of manipulative rules?
>
> Have the common text introductory chapters on sets or functions led to subsequent coherent organization of algebra and geometry, or have the concepts and terminology been learned as sterile formalism and then forgotten?

If one can infer teaching emphasis from testing practice, the syllabus of 1972-73 National Assessment [7] gives some insight into the above questions. After consultation with mathematicians, teacher educators, classroom teachers, and laymen the NAEP staff adopted a content/behavior matrix in the form shown below to guide test development.* The content dimensions include sets,

Content / Behavior	Number and Numeration Concepts	Properties of Numbers and Operations	...	Logic
1. Recall of definitions, facts, and symbols				
2. Perform mathematical manipulations				
3. Understand mathematical concepts and processes				
4. Solving mathematics problems				
5. Using reasoning to analyze problems				
6. Appreciation of mathematics				

* The goals, procedures, and findings of National Assessment will be discussed in detail in Chapter 5.

inequalities, functions, probability and statistics, and logic along with the more predictable conventional topics. But of the 250 items used with 17 year old students, only 15 covered concepts or skills in these topic areas that would not have been likely to appear in a similar effort of the pre-1960 era. Balanced appraisal of the NAEP testing framework hardly suggests a drastically revised secondary curriculum, but it must be remembered that the National Assessment goal was to sample the mathematical accomplishments of all ability groups, not only the college bound.

Junior High School. Curricular innovation at the junior high level is generally unencumbered by college preparatory, Carnegie-unit syllabus traditions. This has made change easier to implement, yet much more difficult to survey. Courses adjusted to students of widely varying ability levels all pass under the title of "general math".

Widely used commercial texts that followed the lead of UMMaP and SMSG experimental material are testimony to an altered junior high curriculum. Students covering these texts encounter the concepts and language of sets, algebraic properties of number systems, non-standard numeration systems, informal properties of number systems, and number theory -- rich preparation for high school study and a striking contrast to pre 1960 texts for grades 7-8. These changes are confirmed by inspection of state and local school objectives and curriculum guides developed throughout the 1960's. Yet the question remains: Did the innovations become regular features of classroom instruction and testing?

NACOME found no firsthand survey data that indicate relative emphasis of new and traditional junior high school topics. Again indirect evidence comes from analysis of National Assessment and commercial standarized test battery syllabi.

Of approximately 200 items used to sample mathematical attainments of 13 year olds, NAEP included very few related to sets (6), probability (6), inequalities (2), and non-decimal numeration systems (2) -- topics that entered seventh and eighth grade instruction recently. There were many geometry items, though stress was on measurement aspects that are not particularly novel. On balance the assessment item pool shows some signs of a changing curriculum, but not dramatic upheaval.

As part of the emerging movement toward accountability for educational programs, both state and local school systems have begun to make use of several nationally standardized tests. Syllabi for those test batteries are developed to reflect common goals of school programs, so it seems reasonable to infer classroom emphasis from the tests. Unfortunately, the four most widely used batteries appear to be measuring quite different kinds of school programs. Set concepts, elementary number theory, and geometry appear in all batteries; but the emphasis varies from slight to prominent and the knowledge tested is most often only use of language and symbolism.

Elementary School. Recent critics of schools have alleged broad decline in achievement of basic reading and mathematical skills, attributing this decline to curricular innovations including "new math". Evidence that will be presented in Chapter 5 supports the contention that test scores are falling. But interpreting this trend as a consequence of specific curricular innovations is less convincing. Primary responsibility for developing arithmetic skills lies with K-6 instruction. Thus this section attempts a thorough analysis of the character and extent of curricular change in these grades.

Elementary school curricular change was slower in developing and more difficult to implement than similar junior or senior

high school innovation. Since each elementary school teacher commonly provides mathematics instruction for only one class of 30 students, the diffusion of any innovation is a massive undertaking. Despite their general willingness to try new curricula and teaching methods, elementary teachers are seldom mathematics specialists and few in-service training programs prepared them to exploit fully the letter and spirit of new curriculum materials. The reconstruction of K-6 mathematics was also dependent on information from psychological studies, including many still unanswered questions about growth of mathematical abilities in young children. This barrier to effective curriculum development was compounded by the limited role of experienced elementary school teachers in development teams.

Judging from inspection of popular textbook series or school curriculum guides, the common elementary program has undergone substantial change in the past ten years. The label "arithmetic" has appropriately given way to "mathematics" as curricula incorporate varying amounts of geometry, probability and statistics, functions, graphs, equations, inequalities, and algebraic properties of number systems. Despite presence in most text series, these topics are most often skipped in favor of more time to develop computational skills that are comfortable to and valued by elementary teachers.

Analysis of National Assessment items for 9 year olds confirms the picture of slow change in the substance of elementary mathematics instruction. Only a few items on sets and twenty-five items on geometry (about half involving recall of names for figures) are exception to emphasis on arithmetic concepts and skills.

The most widely used commercial achievement test batteries vary immensely in the extent to which they test mathematical knowledge beyond manipulative skill. Several attempt to probe student understanding of number and numeration concepts, operations, and properties, though material on probability or functions or reasoning

patterns is not common. If the new test syllabi are compared to an image of K-6 mathematics devoted to arithmetic computation alone, they indicate a noteworthy shift toward comprehension goals -- still falling short of expections in most curricular innovations.

The above collection of information permits only oblique and sketchy inference of "new math" impact on elementary schools. Recognizing the extremely limited data base for decision making in this crucial phase of mathematics education, the National Council of Teachers of Mathematics commissioned a 1975 exploratory survey of second and fifth grade curriculum and instructional practice. Responses from 1220 teachers (in a sample of 3000) help elaborate the scene with data more directly representing classroom activity. [8]

The teachers were asked 120 questions about the content and style of instruction in their classes, their own preparation to teach mathematics, and their feelings and beliefs about priorities for instruction. For instance, they were asked to estimate the relative importance of skills and concepts as instructional goals.

Do you prefer a text that has:

a. extreme emphasis on skills and drill (14%)
b. greater emphasis on skills than on concepts and principles (27%)
c. equal emphasis on both (48%)
d. greater emphasis on concepts and principles than skills (2%)
e. extreme emphasis on concepts and principles (0%)

The teachers were given the following list of content topics and asked to report on whether the topics were in texts and state or local objectives and tests, then to judge instructional time allocated to each, and to indicate how they established time priorities for instruction.

Computation with whole numbers
Concepts of number and operations
Fractions

Problem solving and applications
Measurement
Metric System
Geometry
Graphs and Statistics
Probability
Relation and Functions

Not surprisingly, teachers judged that texts, objectives, and testing programs emphasized the first five content topics. However, 65% reported that their texts contained little or no treatment of the metric system; 45% reported that metric system objectives were not a part of their state or local syllabi: 50% reported metric system not a part of system-wide testing; and 61% spent fewer than 5 periods per year on metric system ideas. The most common explanation for slight attention to metric skills and concepts was lack of suitable instructional materials.

Though geometry is mentioned as being part of texts, objectives, and testing, 78% of the teachers report spending fewer than 15 class periods per year on geometry topics. Graphs/statistics and probability are reported by many teachers as not treated in their texts (52% and 62%), not part of system objectives (40% and 42%), not part of testing (43% and 58%), and taught fewer than 5 periods each year (55% and 74%). Over 45% of the teachers judged probability difficult for students and of low importance, their reasons for small amount of instructional time assigned. While a substantial number of teachers report material on relations and functions in their texts (over 70%), nearly half of the teachers report no testing of these concepts and skills and they devote fewer than 5 class periods per year to the topics.

The impression that emerges from surveying the limited indicators of recent curricular goals is one of modest movement toward inclusion of new mathematical ideas at all levels and of renewed emphasis on understanding the structural unity in mathematical ideas

and methods. The effectiveness of programs aiming at these goals will be discussed in detail in Chapter 5; but many curricular decisions reflect professional value judgments not settled easily by comparative data collection. Almost as soon as UICSM, UMMaP, and SMSG began work, disagreement erupted over proper content and organization of mathematics curricula.

1.3 Analysis of the Innovations

The most persistent and influential criticism along all fronts of recent school mathematics innovation has come from Morris Kline, professor of applied mathematics at New York University. For nearly twenty years Kline has argued eloquently against reform which he claims emphasizes.

- -- novel content such as set theory, Boolean algebra, topology, symbolic logic, and abstract algebra, which is inappropriate for school curricula;
- -- rigorous deductive logical presentation of ideas, contrary to all historical experience of mathematical discovery;
- -- abstract ideas not properly grounded in concrete experience of the physical origins for mathematics;
- -- sterile excesses in terminology and symbolism. [9]

The charges have gained some support among teachers and mathematicians.

Are the main lines of this criticism on target? Do they accurately represent the goals or accomplishments of recent efforts? What redirection of current curricula seems warranted in light of the criticism? Balanced appraisal of Kline's claims is difficult in view of the immense diversity of intentions and accomplishments at various levels of mathematics. Undoubtedly one can locate many instances of such excess in existing text materials and anecdotes of classroom observation. But carefully reasoned judgments of reform efforts can not depend on selected evidence of pathology.

Content Innovations. The criticism of recent content innovations can be analyzed in several ways. First, though several

curriculum projects experimented with teaching concepts from Boolean algebra, topology, symbolic logic, and abstract algebra, no serious proposal has urged dropping the traditional subject matter in favor of these new fields and this change has not taken place in schools. The major content innovations recommended by the Commission on Mathematics and subsequent curriculum projects focus increased attention on fundamental subjects -- geometry, probability, and statistics.

In our view, the introduction of coordinate and transformation methods and solid geometry concepts make the traditional high school course far more useful preparation for application of geometry to future mathematical and scientific study. Attempts to include intuitive geometry in elementary and junior high school programs are equally laudable. But slow acceptance of these topics in new programs and continuing controversy over the content and organization of high school geometry are evidence of fundamental disagreement over the purpose of school geometry as a whole.

The case for early and extensive school probability and statistics instruction grows stronger each year as demands for statistical decision making spread throughout all areas of contemporary society.

Though Kline holds little brief for importance in the school curriculum of algebraic concepts like inequalities, matrices, or groups, we disagree. Numerous problems of optimization in the complex systems of social and management science demonstrate the growing importance of inequalities and matrix methods. Algebraic structures like groups and fields can also be justified by emerging applications at a high level of science, but the case for inclusion of these topics is interwoven with pedagogical issues too.

The popular image of "new math" content innovation centers on set concepts, algebraic commutative/associative/distributive

properties, and number bases. Being most unlike traditional school mathematics, these topics were often featured at exhibitions of "new math" for parents, and they did find substantial space in both experimental and commercial texts. Debate over the merit of these topics has, in our view, taken too little care to distinguish between ends and means of curricular experiences. The most convincing arguments for including set concepts and terminology in school curricula rest on the role that these ideas can play in organizing other mathematical facts and methods -- in demonstrating connections between apparently different mathematical tools so that acquistion, retention, and transfer can be enhanced. Set theoretic ideas offer another vehicle for explaining, illustrating, and practicing basic mathematical concepts and skills. A similar argument supports inclusion of algebraic field properties in study of number systems.

The subtle function of these unifying concepts was often poorly incorporated by new curriculum materials and by classroom teachers. Early modern textbooks began with chapters on set concepts, language, and operations; but the balance of these books used little more than the notation of sets. For example, some teachers began "taking off points" for answers not enclosed in curly brackets. They drilled young children in correct spelling of the "commutative", "associative", and "distributive" terms rather than useful application of the concepts in computation. The proposed means to deepen understanding of useful mathematics became ends in themselves.

Experience with these new ideas over several years of classroom implementation has now led to a much better perspective in both texts and teaching. Furthermore, so-called "second round" curriculum development projects of SMSG, the Secondary School Mathematics Curriculum Improvement Study (SSMCIS), and the Comprehensive School Mathematics Program (CSMP) are generating promising approaches for introducing the concepts and new curriculum structure which capitalizes on their unifying power.

Role of Deduction. The second major criticism of "new math" innovation is overemphasis on deductive presentation of ideas. We cannot deny the contention that historically most mathematical ideas developed intuitively out of physical analogies and generalizations based on simple cases -- that formal logical organization is an end product of inductive exploration and analysis. But the transformation of this view into telling criticism of recent school mathematics innovations fails on at least three counts.

First, the new programs have by no means adopted a uniform deductive presentation of ideas. Analyzing the elementary texts of SMSG, E. G. Begle [10] has convincingly refuted Kline's charge of pedagogy ruled by logic. At the high school level, the traditional geometry course experience in deductive reasoning has been reorganized to give a more realistic view of the role played by axioms and formal definitions. In contrast to the earlier pattern of listing all axioms, postulates, and definitions at the outset, new geometry texts introduce assumptions and terms throughout the course only when it can be demonstrated that further inference depends on them.

Newer programs do give emphasis to deductive reasoning in algebra I, algebra II, and trigonometry, but the rationale for this emphasis is not based on naive misunderstanding of mathematical methods. It is instead grounded in the hypothesis that students perceiving the structure of mathematical ideas will become more effective learners and users of the subject. The deductive organization of facts and methods in number systems or algebra is only one of several vehicles for conveying structure.

A third weakness in Kline's criticism of logic in school mathematics is his reliance on the creative experience of great mathematicians for insight into proper pedagogy for all students. Most of our students seek from mathematics a collection of well established concepts and methods that can be applied to problems outside of

mathematics. It is not at all clear to us how organization of these tools into logical structure will be necessarily a barrier to learning -- though in many cases initial presentation might well be guided by psychological rather than logical organization.

No doubt some recent curricula and instructional trends have over-emphasized or misunderstood the proper interaction of inductive and deductive methods in mathematics. Yet logical reasoning is one of the fundamental characteristics of mathematical thought and a crucial contribution to problem solving. Ignoring logical structure in school mathematics would be a serious mistake.

Role of Abstraction. A curriculum organized by broad structural concepts and processes is predicated on the ability of learners to make appropriate abstractions and generalizations. These abstractions arise from, but ultimately transcend the limits of, specific concrete representations. Addition of numbers is originally suggested by combining sets of blocks or sticks or beads; properties of measurement are observed in constructing rulers, protractors, or balance scales; group theorems are suggested by similarities and differences among whole number, integer, and rational number operations.

Proper timing and balance of concrete and abstract experiences has been a fundamental pedagogical problem in recent curriculum development. Kline argues that new programs consistently err in the direction of premature abstraction; but again, Begle has analyzed Kline's charges in light of SMSG text material and found them to be unwarranted. [11] The basic thrust of K-6 innovation has been to replace traditional rote learning of formal arithmetic with meaningful development grounded in physical experience. With this background, secondary level courses appear less forbiddingly abstract.

Current school mathematics curricula can probably be improved by more creative interplay of concrete experience and abstract ideas.

Several "second round" projects have shown ways that this development might proceed. Reaction against abstraction at any level denies the very real contributions that its process and product make to mathematics and would be a step backward in developing improved school programs.

Role of Symbolism and Terminology. One prominent manifestation of abstraction in mathematics is the symbolism used to convey ideas. Since early Babylonian and Egyptian efforts 5000 years ago, mathematicians have groped for even more efficient numeration systems to record quantitative information. It was an attempt to improve teaching about numeration that led to one of the best known features of "new math" -- number bases.

Research on arithmetic difficulties of young children shows that inadequate understanding of our decimal place-value numeration system is one of the most common barriers to improved computation. Reasoning that work with other numeration systems would highlight, by contrast, the essential features of base 10, many curricula introduced extensive activity with base 2, base 3, Egyptian hieroglyphics, and so on. Research has since shown this practice to be relatively ineffective and already emphasis on "bases" is justifiably declining in elementary and junior high school programs. A similar appropriate fate has befallen the overly rigorous distinction between mathematical objects and their names, most notably the number/numeral fetish. However, the de-emphasis on both topics must not result in a loss of the insight that can be gained by teachers and students through proper understanding of the principles involved.

In a move to clarify the vague language of many traditional mathematics texts and instructional traditions, most new programs strove for precise use of symbols and definitions of terms. The effort did not result in any kind of uniform symbolic conventions or definitions. For instance, different texts denoted angles and

their measures with a bewildering array of symbols: $\angle A$, $\measuredangle A$, $m\angle ABC$, $\overset{\frown}{ABC}$, etc. And there is still lively debate over the preferred definition of angle: union of concurrent rays, intersection of half-planes, or rotation. Nonetheless, Kline and other critics have argued that these efforts at precision have no practical or pedagogical payoff; they argue that symbols and terminology do not make understanding -- in fact, excessive formalization is a barrier to learning. Most recent trends in school texts and teaching practice indicate acceptance of that view and return to a much more modest emphasis on symbols and precision of expression. We only caution that historically the precise symbolic language of mathematics has played an important role in the development and communication of ideas. The most effective instructional balance of informal and rigorous expression is a function of mathematical topic, student aptitude, and student experience. Understanding of this interaction merits extensive psychological and classroom research.

Furthermore, in a world of increasing complexity weighed down by a flood of concrete and detailed information, understanding and skill with processes of abstraction and symbolic representation may be as important in the ordinary citizen's equipment for life as were the purely numerical skills (still not to be neglected) in an earlier era. Margaret Mead, in Culture and Commitment, and Marshall McLuhan, in The Medium is the Message, are but two of the many voices which claim that our young people live in a radically different culture that we constantly fail (or may be unable) to understand. Characteristic of this world is the constant bath of "non-linear" multisensory symbols. General familiarity with symbols -- their uses, their formalities, their limitations -- is an educational objective appropriately developed in the mathematical area of school life along with the direct and unique experiences in abstraction and generalization which the subject provides. These elements should never be completely abandoned in any particular program, but developed and fostered in an appropriately proportioned manner.

Summary. The preceding analysis has focused on criticism of innovation planned or accomplished in recent curriculum development. From a 1975 perspective the principal thrust of change in school mathematics remains fundamentally sound, though actual impact has been modest relative to expectations.

The content innovations K-12, the emphasis on student understanding of mathematical methods, the judicious use of powerful unifying concepts and structures, and the increased precision of mathematical expression have made substantial improvement in the school mathematics program. Unfortunately, the innovations have not fulfilled the euphoric promise of 1960, and current debate seems intent on locating blame for failures in real or imagined "new math" programs. Popular reports of this debate suggest inevitable and bitter polarization of the mathematics community on the issues:

old or new
skills or concepts
concrete or abstract
intuitive or formal
inductive or deductive

This dichotomization of curricular issues does not accurately convey the intentions or the accomplishments of recent innovations. Furthermore, the acrimonious criticism sends many teachers and laymen in retreat to truly outmoded curricular goals, rather than moving forward by building on positive features of recent change to meet the new mathematical challenges of 1975 and beyond.

Much of this wasted energy and unnecessary battling can be attributed to a fallacy that seems very difficult to eradicate: that of viewing the "new math" as a monolith, a single phenomenon that one can be for or against. Actually it refers to two decades (1955-1975) of developments that had a general thrust and direction but sprang from many roots, took many different and even opposing forms, evolved and changed with facets disappearing and new ones arising. Using the phrase "New Math" to describe this era gives

it the nature and flavor of such phrases as "The Roaring Twenties" or "The Great Society".

NACOME strongly recommends that all who are interested in what is going on in school mathematics today, whether as supporters or critics -- parents, teachers, mathematicians, educational administrators, lawmakers -- from this point in time use the term "new math" only as an historical label for the vague phenomenon or the very diversified series of developments that took place in school mathematics between 1955 and 1975. Reference to current school mathematics, its status, its trends, and its problems should be made only in such common-noun terms as the "present mathematics program", "current school mathematics", "contemporary mathematics teaching", etc.

CHAPTER 2. CURRENT PROGRAMS AND ISSUES

Mathematics program improvements of the "new math" 1960's were primarily motivated and designed to provide high quality mathematics for college capable students -- particularly those heading for technical or scientific careers. Guidance in the curriculum development came largely from university and industrial mathematicians, and the model for curriculum structure was the logical structure of mathematics. Today mathematics curriculum development focuses on issues largely ignored in the activity of 1955-1970. Responding to the concerns of classroom teachers, as well as educators and laymen interested in the basic goals of general education, attention has now shifted to programs for less able students, to minimal mathematical competence for effective citizenship, to the interaction of mathematics and its fields of application, and to the impact of new computing technology on traditional priorities and methods in mathematics. Furthermore, the dominant role of mathematical structure in organizing curricula has been challenged by many who advocate pedagogical or psychological priority in determining scope and sequence.

2.1 New Curricular Emphases

While each of the above topics (including technology, if not computers) has been a major issue at some time in the history of mathematics education, the first section of this chapter is devoted to a perennial problem once again on center stage -- the relative importance of "basic skills."

Computational Skills. One of the strongest undercurrents in present curriculum decision-making is pressure to re-emphasize computational skill in arithmetic and algebra. The trend is undoubtedly a response to public criticism of declining computation test scores. However, NACOME finds the case for such return to skill oriented curricula completely unconvincing. In the first place, test score evidence which we have examined does not suggest precipitous drop in mathematics achievement attributable to the content and emphases of "new math". The data are presented and discussed in Chapter 5 of this report. In general our impression is that mathematics performance only paralleled, and in some cases resisted, declining performance in all school subjects. Moreover, it appears to us that the case for decreased classroom emphasis on manipulative skills is stronger now than ever before. Impending universal availability of calculating equipment suggests emphasis on approximation, orders of magnitude, and interpretation of numerical data -- not drill for speedy, accurate application of operational algorithms. In higher level manipulations of algebra, computers are beginning to demonstrate new ways of approaching traditional equation solving problems and the symbolic manipulations that require so much instructional time in secondary school.

Conceptual thought in mathematics must build on a base of factual knowledge and skills. But traditional school instruction far over-emphasized the facts and skills and far too frequently tried to teach them by methods stressing rote memory and drill. These methods contribute nothing to a confused child's understanding, retention, or ability to apply specific mathematical knowledge. Furthermore, such instruction has a stultifying effect on student interest in mathematics, in school, and in learning itself.

Far from ignoring functional competence in basic computational skills, many teachers of the "new math" era sought levels of skill achievement that had escaped traditional efforts. But they sought

improved skill performance through deeper student understanding of the structures underlying computational methods. Though the goal of increasing computational competence has not been reached on any massive national level, this failure does not invalidate the "understanding leads to skill" hypothesis. We have reason to suspect that in many classes teachers very poorly related structural understanding to algorithms embodying the structures. In other classes, teachers made structure a royal road to skill and failed to provide any emphasis on computational practice. The conviction remains that a reasonable understanding of number and process, supported by necessary elements of practice, can lead to effective computation, while mere return to rote memorization and drill cannot.

The members of NACOME view with dismay the great portion of children's school lives spent in pursuing a working facility in the fundamental arithmetic operations. For those who have been unsuccessful in acquiring functional levels of arithmetic computation by the end of eighth grade, pursuing these skills as a *sine qua non* through further programs seems neither productive nor humane. We feel that providing such students with electronic calculators to meet their arithmetic needs and allowing them to proceed to other mathematical experience in appropriately designed curricula is the wisest policy.

Applications. Mathematics and science have had a long and tremendously productive interaction -- science providing the genesis of most important mathematical ideas, and mathematics in turn providing the intellectual tools for description and prediction of scientific phenomena. The Commission on Mathematics urged strong secondary school programs to meet increasing mathematical demands of scientific careers. But in 1959 this emergence of mathematized biology, psychology, sociology, and management was only beginning at a high level. Since specific implications and illustrations for school curricula were not evident, the first generation experimental texts did not incorporate extensive applications as motivation for learning or as

practice in the modeling process. Inspection of current commercial texts, standardized tests, national assessment items, or state and local mathematical syllabi confirms the disappointing impression that "application" in school mathematics means "word problem." For most of these problems, the main task for students is translating the technical jargon of mathematical prose into simpler language and then into suitable symbolic form. Furthermore, subsequent arithmetic and algebraic manipulations inevitably lead to simple closed-form solutions which students quite accurately see as applying to few realistic situations.

For years this weakness in the practical problem solving phase of mathematics teaching has been consistently and broadly criticized, most notably by Morris Kline. Recommendations on the appropriate interaction of school mathematics and science have been forcefully presented in a sequence of curriculum planning conferences.* Although there is general agreement that ability to apply school mathematics is a crucial learning goal K-12, efforts to achieve this goal take widely divergent and contradictory forms:

> Should applications be used mainly to introduce or illustrate mathematical ideas that have been sequenced according to the logical structure of mathematics?
>
> Should the scope and sequence of mathematics instruction be designed to meet the needs of parallel science instruction?
>
> Or should the entire school curriculum be reorganized around broad interdisciplinary problems, with mathematics developing incidentally?

At the elementary level, three curriculum development efforts stand out as explorations of the way that mathematics instruction

*Cambridge Conference on the Correlation of Elementary Science and Mathematics (1967), Cape Ann Conference on Junior High School Mathematics (1973), Estes Park Conference on Learning Through Investigation and Action on Real Problems in Secondary Schools (1973), Snowmass Conference on K-12 Mathematics Curriculum (1973).

might interact with applications. First is the Minnesota Mathematics and Science Teaching Project (MINNEMAST) begun in 1961. The goal of MINNEMAST was production of coordinated science and mathematics materials, and at termination of the project in 1970, twenty nine units for grades K-3 were available -- though not widely used. Because there was no summative evaluation of the MINNEMAST program it is difficult to assess its effectiveness or to infer feasibility of the integrated math/science curriculum style.

A second major elementary project grew out of the 1967 Cambridge Conference. Unified Science and Mathematics for Elementary Schools (USMES) aims to develop student ability to make informed decisions on matters affecting their own lives and society in general. The vehicle for attaining this instructional goal is a collection of challenging real but manageable problems to be solved by elementary school students. The problems are generally interdisciplinary and it is intended that their solution will lead to learning of concepts and methods from science, social studies, and mathematics as well as heuristic strategies applicable in any decision-making situation. The USMES goal is production of 32 problem units, 4 for each grade 1-8. Evaluation completed during the 1974-75 school year indicates that these problem solving experiences are attractive to students and teachers, who see many important mathematical ideas developed by the units. However, the extreme diversity in patterns of trial usage make it difficult to judge the potential of USMES as more than a supplement to ordinary mathematics and science curricula. If anything, USMES teachers have replaced regularly scheduled science time, not mathematics. Progress toward the goal of enhanced student problem solving ability is very difficult to assess.

The third type of curriculum development relating mathematics and applications at the elementary level is Project ONE. This project differs from MINNEMAST and USMES in its expressed commitment to produce what is basically a mathematics program. However, by

focusing its objectives on the requisites for quantitative thinking in realistic situations, Project ONE proposes a breakdown of the traditional sequential logical organization of school mathematics. Instead it offers a series of 65 half-hour television programs for use in classrooms and at home (with coordinated hands-on materials) organized first to meet the needs of student interest and relevant problem solving activity. The underlying mathematical themes will be counting and ordering, measurement, estimation, ratio and size scaling, and graphing. These content objectives will be paralleled by goals for developing broad heuristic strategies. Clearly the project aims to use applications to motivate and illustrate mathematical learning; but the criterion of usefulness also appears to play a strong role in selection and sequence of mathematical topics. The programs are scheduled to be televised beginning in January 1976, so it is impossible to estimate their effectiveness at this time.

At the junior and senior high levels, mathematics and science instruction have traditionally fitted into separate courses structured according to the patterns of parent disciplines and taught by content specialists. Thus it is not surprising that in grades 7-12 most energy has been directed toward supplementing mathematics materials with appealing motivational and illustrative applications. SMSG produced three such resource volumes for teachers; an American Statistical Association/NCTM Committee on the Statistics and Probability Curriculum has produced a four volume resource series Statistics By Example; and a joint NCTM/MAA committee is developing a Sourcebook on Applications of Mathematics. Research for Better Schools (RBS) has produced learning activity packages illustrating the types of mathematics used in a wide variety of non-scientific occupations. In addition to the resource books, SMSG produced and field tested several "mathematics through science" units for secondary schools. The impact of these resource efforts is difficult to assess. It is safe to say that they deserve to be much more widely available and known to teachers.

There are at least five current curriculum development projects illustrating major new ways to blend mathematics and applications at the secondary level. Following the lead of the Cape Ann Conference (1973), a project at Boston University is producing seventh and eighth grade mathematics courses which will emphasize mathematical thinking in a context of applications. From an orientation apparently similar to that of Project ONE, this curriculum will stress: 1. Orders of magnitude and estimation; 2. Functions, graphs, and their applications to life sciences as well as physical sciences; 3. Descriptive statistics; and 4. Space perception and representation. Quite evidently the infusion of an applications point of view is influencing selection and organization of mathematical topics.

Two other "applications oriented" projects are developing alternatives to the traditional ninth grade algebra course The UICSM Introduction to Mathematical Methods in Algebra, Geometry, and Probability and Zalman Usiskin's First Year Algebra Via Applications attempt profuse practical illustration of the concepts of algebra and probability; more important, the materials try to frequently develop important mathematical ideas as models of physical problem situations.

The Engineering Concepts Curriculum Project (ECCP) has produced a senior high course, The Man Made World (TMMW) emphasizing use of the systems approach to solve social, political, environmental, and technological problems. The course is interdisciplinary and not intended to replace any existing mathematics course. But the problems draw on many mathematical methods and several chapters are devoted to design and application of computers. Many high school teachers are finding TMMW a rich resource of ideas for standard mathematics courses. The material is designed for students of middle range ability and has special appeal for inner city students. Modifications of the text for use in an activity based junior high school setting are now available.

Quite a different setting for mathematical learning is emerging from the Biomedical Interdisciplinary Curriculum Project (Biomed). This program for high school juniors and seniors is organized around concepts and problems from medicine with topics from science, social studies, and mathematics chosen as they contribute to the career oriented educational goals of students interested in the health sciences.

Of these secondary school mathematics/applications curricula, only The Man Made World has been completed and fully field tested so far. However, the range of developing alternatives offer a genuine challenge to secondary curricula whose topics are traditionally selected and organized according to the logical structure of pure mathematics.

At elementary, junior high, and senior high school levels, efficacy of various application oriented curricula can and should be judged by evaluation of these innovative programs.

> Do the applications improve student interest or attitudes?
>
> Does experience with applications improve understanding of mathematical ideas or problem solving ability?
>
> Do interdisciplinary programs place too much faith in incidental learning of mathematics?
>
> Which mathematical concepts and skills seem most effectively taught through specific real life models?

While the verdict is still out on these fundamental questions we urge teachers at all levels to experiment judiciously with available materials and thus participate in developing balanced mathematical experiences for their students. Meanwhile, both the extensive claims for the meaningfulness of applications in the curriculum and the volume of activity taking place to develop them point to the necessity of early and serious evaluative efforts to answer some of these questions.

Many proponents of the application based programs seem to suggest need for choice between antithetical goals: applications or theory, problem solving or structure building. We are convinced that this is a false antithesis, that these aspects of mathematical education can and should complement each other. As Peter Hilton has suggested, "if we are to be able to apply mathematics, there must be some mathematics to apply" and "problems are solved by recognizing the structure and nature of those problems, and this recognition comes by placing the problem firmly in its appropriate analytical context." [1]

It is also important that the term "applications" not be construed too narrowly. Applications include all of the practical uses of mathematics needed by everyone for daily life today and in the future, a need mathematical educators accept as both valid and yet poorly fulfilled by present school mathematics programs. But applications in school mathematics cannot be limited to this important dimension. They must include all problems susceptible to mathematical analysis -- not only those encountered by every citizen in his daily life. Applications must also include the development and use of sophisticated mathematical models in the sciences and other scholarly fields.

Curricula for Less Able Students. The original SMSG secondary school courses were designed for college capable students. But several subsequent investigations indicated that by slowing the pace of instruction the same ideas could be learned as well by less able students. For a short time general mathematics texts incorporated many of the content innovations of more high powered courses and reports of success were common. The practice of offering Algebra I as a two year course (grades nine and ten) emerged and is now quite widespread. Nonetheless disillusion with "new math" for low ability students soon set in. There was public dismay at reported student inability to perform practical arithmetic. There were repeated calls to develop more appropriate curriculum materials.

Development projects responding to this call have focused mainly on pedagogical innovations to meet the special needs of slow learners -- variety of activity, physical embodiment of ideas, low reliance on reading, more practice with skills, motivation by practical utility of skills, etc. As a result, logical structure has taken a back seat to pedagogical possibilities in determining curriculum content. For instance, UICSM produced a year long seventh grade course, Stretchers and Shrinkers, devoted solely to fractions and operations; the companion eighth grade course was all Motion Geometry. An NCTM committee produced the series of booklets for slow learners, Experiences in Mathematical Discovery. The booklets are largely independent of each other in content, with topics chosen because of practical importance and mathematical appeal to slow learners.

Students having difficulty with any subject are the first to ask "What good is all this stuff?" So renewed attention to low ability students has strengthened interest in applications. Most such development -- specifically directed to non-college bound students -- has been generated in local school system and commercial textbook projects. For instance, Baltimore County Public Schools have produced modularized applied mathematics courses treating consumer, vocational, and technical topics such as renting an apartment, auto mechanics, machine shop, home decoration, etc. There are similar course materials available in many parts of the country -- for example, RBS in Philadelphia, COLAMDA in Denver, and WYMOLAMP in Wyoming. Recently quite a number of commercial texts have appeared to meet the demand for high school consumer and vocational-technical mathematics courses.

The NCES survey data for 1972-73 indicate growing offerings and enrollments in courses intended for non-college bound students.

Table 2

U. S. Public Schools Offering Various Mathematics Courses Given as a Percent of Total Number of Schools [2]

	1960	1972
Remedial Math 7-8	--	9.7%
Remedial Math 9-12	--	19.4%
Consumer Math	--	2.9%
Applied Math	--	6.8%

The 1972-73 enrollment in these courses and high school general mathematics was approximately 2,750,000 students, with every reason to believe the number has increased since that survey year.

The shift of labels from "the less able" to "the non-college bound" in the preceding paragraphs is symbolic of the ever-shifting nature and description of certain target populations in mathematics teaching. In its inception, "non-college bound" was synonymous with "less able." But changing cultural conditions have led increasing numbers of students from all levels of ability and high school success to become "non-college bound", to seek a direct entry into adult life and work after high school. The general literacy of a TV-oriented age and on-job training opportunities in many reasonably high level occupational areas make such a move feasible for these young people and acceptable to many educators and industries. Similarly, various mathematical education programs have been aimed at target populations described as "disadvantaged" or "inner city". The loose and fluctuating use of these labels has been its own kind of problem. There are "disadvantaged" youngsters in many places beyond the inner city. Neither the disadvantaged nor inner-city populations are necessarily less able; certainly many programs work directly at increasing the college bound numbers from these populations, rather than capitulating to creation of programs which type them as non-college bound.

The growing interest in applications, in programs for low ability students, in the immediate employment needs and upward mobility

thrusts of inner city populations and other disadvantaged groups are all reflected in recent developments of career-oriented curricula The broad term "career education" has many definitions. When interpreted as a collection of career awareness activities, it seems to offer valuable enrichment and motivation for current programs. But if, as some suggest, career education implies reorganization of the entire school curriculum around preparation for the demands of 15-20 job clusters, there are serious implications for each of the traditional disciplines.

The challenge of career education proposals and the more general search for curricula appropriate to students not interested in collegiate level scientific and technological education has provoked lively debate on the proper goals for school mathematics instruction. Contrasting syllabi and texts for the different student audiences, one is quickly struck by the emergence of two distinct mathematical cultures -- one focusing on broad structural concepts and heuristic methods, the other on computational skill and specific techniques applicable to problems of everyday life and specialized trades. Is this dichotomy inevitable and appropriate? Are there readily identified minimal mathematical competencies for every secondary school student to acquire? Is narrow, career-oriented mathematical training a suitable or effective alternative to the traditional programs for less able students? These are fundamental questions that deserve careful study by all concerned professional groups.

Computers. Curriculum innovation of the 1960's could only anticipate the educational impact of emerging electronic calculating and computing technology. Though computers were then in widespread scientific and commercial use, school access via time-sharing or on-site computers was extremely rare. Thus computer capabilities influenced "new math" only in minor or indirect ways.

In 1975 student access to computers is growing, and the problem solving and instructional potential of computers has become

much clearer. The American Institutes for Research (AIR) survey of Computing Activities in Secondary Education (1975) [3] indicates that over 58% of American secondary schools now make some use of computers (up from 34% in 1970). Over 26% of these schools use the computer to some extent for instruction (up from 13% in 1970), and of the instructional uses 43% are in mathematics (down from 47% in 1970).

Computers can influence the content and process of mathematics education in four basic ways:

1. Mathematics teachers are frequently called upon to provide basic instruction in principles of computer science. This includes computer literacy -- structure, capabilities, and limitations of computers -- and programming in various special languages.
2. Carrying out high speed arithmetic calculations and logical operations according to given directions, computers assist problem solving, exploration of mathematical concepts, and simulation of complex systems.
3. Communicating with students via teletype and a pictorial display mechanism, computers can serve as a medium of instruction -- presenting information, conducting drill and practice of skills, and engaging students in dialogue about mathematical ideas -- all generally referred to as computer assisted instruction (CAI).
4. For management of instruction, computers can keep records of diagnostic testing, prescribe instruction, and monitor student achievement -- all generally referred to as computer managed instruction (CMI).

The 1975 AIR survey noted above indicates the relative frequency of these four instructional computer applications in mathematics:

computer science*	20.6%
problem solving and simulation	55.3%
CAI	20.8%
CMI	3.3%

*Computer science usage is reported in a separate category also, constituting 21.8% of all instructional computer use in schools.

In mathematics the median number of students per school involved with the computer is 30, likely indicating a select group of advanced mathematics students. Though some high school texts now include independent computer programming units, our survey of state mathematics objectives revealed only a few modest efforts to incorporate computer goals. Standardized tests and the National Assessment of 1972-1973 include no items related to flow charts or computing, though the planned 1977-78 National Assessment will probably include computer literacy items.

The rapidly expanding availability of computers in schools and the equally rapid improvement in capabilities of the machines themselves mandates thorough re-examination of the content and methods of mathematics instruction at all grade levels. The nearly universal experience of mathematics teachers introducing their students to computing is a strong boost in student motivation and interest. There have been particularly noteworthy results with students identified as educationally disadvantaged. The demands of computer programming reinforce desirable mathematical methods such as organizing information, analyzing procedures systematically, checking answers for reasonableness, and finding errors. Furthermore, computer access increases student power to explore mathematical concepts by checking many examples in search of a pattern.

But computer access suggests more fundamental rethinking of curriculum priorities and organization. If a computer system contains routines for solving quadratic equations or systems of linear equations, should algebra students still spend considerable time practicing formal techniques of solution? If computers can generate precise geometric drawings in appropriate three dimensional perspective, given only a rough hand sketch, must students devote extensive time to descriptive geometry? Should probability instruction emphasize sophisticated combinatorial counting techniques or computer-based Monte Carlo modeling? Are common fractions worthy of extensive

school practice when the language of computers is decimals? Should the curriculum pay more attention to errors and approximation since computers inevitably truncate or round off all calculations?

Computer capabilities also open up fascinating possibilities for realistic application of mathematical techniques and simulation of interdisciplinary problem solving situations. Because computers can easily cope with complex calculations and numerical values, once programmed by a human problem solver, it seems likely that teachers and students will be able to focus more attention on the modeling process -- problem formulation and interpretation of results -- and less on practicing manipulative skills and accuracy.

Efforts to explore the many potential computer contributions to mathematical learning have been more diverse and less influential than the major curriculum projects such as SMSG or UICSM. A small set of formal projects have developed around existing computer facilities. But a great deal of work has also been generated by local school efforts, communicated around the country by a lively network of user newsletters.

The major curriculum development project that has explored computer enriched learning of mathematics is the Computer Assisted Mathematics Project (CAMP) at the University of Minnesota. CAMP produced a series of secondary school books (grades 7-12) to be used parallel to existing curricula. The CAMP texts demonstrated ways that flow charting and programming of basic mathematical techniques could elaborate and deepen instruction.

Efforts to replace existing mathematics courses with programs depending on access to computers -- approaching traditional topics from novel viewpoints -- have not been prominent. A small consortium of Denver area schools participated in development of A Second Course in Algebra and Trigonometry with Computer Programming. At the level

of calculus, the Florida State CRICISAM project has reorganized conventional presentation of elementary calculus in order to take advantage of computer oriented activities as part of basic concept formation.

Two major projects are currently exploring much more daring reconceptualizations of the school mathematics curriculum, fully exploiting computer capabilities. At the elementary level, Seymour Papert of MIT Project LOGO is investigating the consequences of designing mathematics instruction predicated on an environment of computers and computer controlled devices. Papert argues that computer access should alter our conceptual approach to basic ideas of geometry and number and also offer a fertile environment for teaching general problem solving strategies.

At the secondary level, the University of Pittsburgh's Project SOLOWORKS has explored a similar point of view on computing and mathematics teaching. Principal investigator Thomas Dwyer argues that computers can provide the vehicle for a learning style with powerful motivational and cognitive effects if students develop SOLO ability at programming and then apply their ability to challenging interdisciplinary problems. Dwyer claims that a loosely structured matrix of appropriate problems -- not a linear hierarchy of specific mathematical skills -- is the style of curriculum organization that will most effectively stimulate powerful mathematics learning.

The SOLOWORKS project, PLATO at the University of Illinois, and Huntington II at SUNY-Stony Brook have developed extensive computer-based simulation activities which give exciting promise of fulfilling that objective for computer use. For instance, both PLATO and Huntington II systems include models of population growth which allow students to observe the effects of changing birth and death rates on the age profile of populations in a country. One Huntington II simulation activity helps students experience the

interaction of competitive forces in an economic system. A fascinating PLATO simulation allows students to "conduct" research by probabilistic computer simulation of the breeding of fruit flies. These projects and several others are also involved in research and development of what might more properly be labelled computer aided instruction or computer mediated instruction.

In 1972 a CBMS committee made six major Recommendations Regarding Computers in High School Education [4]. The Committee called for:

1. Preparation of a junior high school course in "computer literacy".
2. Preparation of text materials for follow-up courses in computing, modules which integrate computing into high school mathematics courses, and other modules which utilize computers in simulating the behavior of physical or social phenomena.
3. Development of special programs for high school students showing unusual aptitude and promise in computer science.
4. A major effort aimed at making vocational computer training more generally available and at the same time improving the quality of such training.
5. Development of a variety of programs for the training of teachers of high school courses involving computers.
6. Establishment of a clearinghouse for information about high school computer education.

Each of these recommendations is still important today. Computer literacy has not yet become a prominent goal of school mathematics instruction. But whether it enters as a special junior high school course or as a strand woven into the curricula of mathematics and related school subjects, general understanding of the capabilities and limitations of computers must become part of everyone's education. Contact with computers must expand beyond a few very able mathematics students. The development of modules that integrate computing and regular mathematics courses is a sensible first step

in realizing the extensive impact of technology on the methods of mathematics. However, it is now timely to begin reconstruction of school mathematics curricula, choosing content and organization predicated on universal availability of computers in the near future.

For several years to come mathematics teachers will very likely be expected to carry the burden of computer science teaching at the school level. Thus computer science education must become part of the preparation of mathematics teachers. At least one course should be of sufficient depth to guarantee that the potential teacher can program in a problem oriented computer language well enough to teach it to high school students, and another should cover the broad issues involved in computer literacy.

In school systems that have a combination of access to machines and knowledgeable teachers there has been a proliferation of local curriculum development efforts. But computer materials do not "transport" easily. A center that could coordinate the documentation and dissemination of these existing materials could be a valuable step toward much broader school usage of computers.

Calculators. Nearly everyone realizes that computers are widely used in government, science, business, and industry; but as a topic for K-12 instruction, computing still has the image of an exotic luxury, not high on the financial priorities of hard pressed school budgets. The capabilities and widespread availability of hand-held calculators cannot be so easily ignored on the school level. For under $20 students can obtain a dependable tool to perform the operations of arithmetic that have long been the focus of elementary school mathematics instruction. For under $50 high school students can obtain calculators that handle all the computational functions of any ordinary college preparatory mathematics curriculum. It is extremely difficult to convince students, who have ready access to calculators, that they must develop facility in mental calculation

assisted only by pencil and paper. The simple arguements "What if the calculator is not available when you need it?" or "What if the calculator breaks down?" are justifiably weak motivation for elementary and secondary school arithmetic students.

The challenge to traditional instructional priorities is clear and present. If mathematics education takes full advantage of the new technological capabilities, we envision at least the following kinds of change in school programs. First, the elementary school curriculum will be restructured to include much earlier introduction and greater emphasis on decimal fractions, with corresponding delay and de-emphasis of common fraction notation and algorithms. This change is appropriate to match the language of instruction to the language of calculators (decimals). But further justification lies in the impending metric system adoption, the opportunity to exploit natural similarities between decimal fraction and whole number algorithms, and the chance to avoid very difficult (and less effectively taught) procedures involving common fractions (lowest common denominator, equivalent fractions, and reduction to lowest terms).

Second, while students will quickly discover decimals as they experiment with calculators, they will also encounter concepts and operations involving negative integers, exponents, square roots, scientific notation and large numbers -- all commonly topics of junior high school instruction. These ideas will then be unavoidable topics of elementary school instruction. For instance, students may discover from the calculator that the product of two negative numbers is a positive number and computational facility with integers (using the calculator) will precede, rather than follow, the careful conceptual development of these ideas.

Third, arithmetic proficiency has commonly been assumed as an unavoidable prerequisite to conceptual study and application of mathematical ideas. This practice has condemned many low achieving

students to a succession of general mathematics courses that begin with and seldom progress beyond drill in arithmetic skills. Providing these students with calculators has the potential to open a rich new supply of important mathematical ideas for these students -- including probability, statistics, functions, graphs, and coordinate geometry -- at the same time breaking down self-defeating negative attitudes acquired through years of arithmetic failure.

Fourth, for all students, availability of a calculator does not remove the necessity of analyzing problem situations to determine appropriate calculations and to interpret correctly the numerical results. The user must still determine which calculator buttons to push. With de-emphasis on the purely mechanical aspects of arithmetic comes an opportunity to pay close attention to other crucial aspects of the problem solving process and to treat more genuine problems with the "messy" calculations they inevitably involve. Facility in the mental estimation of arithmetic results, to check that one's calculator is functioning well and that correct problem analysis has preceded calculation, will continue to be useful.

Fifth, present standards of mathematical achievement will most certainly be invalidated in "calculator classes". An exploratory study in the Berkeley, California public schools indicated that performance of low achieving junior high students on the Comprehensive Tests of Basic Skills improved by 1.6 grade levels simply by permitting use of calculators.[5]

Despite the obvious promise of calculators for enriching mathematics instruction, important questions of their optimal use must be investigated by thorough research:

> When and how should calculator use be introduced so that it does not block needed student understanding and skill in arithmetic operations and algorithms?
>
> Will ready access to calculators facilitate or discourage student memory of basic facts?

For which mathematical procedures is practice with step-by-step paper and pencil calculation essential to thorough understanding and retention?

What types of calculator design -- machine logic and display -- are optimal for various school uses?

What special types of curricular materials are needed to exploit the classroom impact of calculators?

How does calculator availability affect instructional emphasis, curriculum organization, and student learning styles in higher level secondary mathematics subjects like algebra, geometry, trigonometry, and calculus?

As yet there is very little published research on these questions -- though we suspect a great many studies are under way. One thing is certain. Calculators will very soon be a tool available to and used by every American. They in no way diminish the importance of school mathematics instruction, but instead allow students to feel the power of mathematics and free time for teachers to concentrate on the conceptual aspects of the subject which are of fundamental importance.*

Metric System. For years mathematics teachers have been attracted to the elegant simplicity of the Systeme International d'Unites (SI), known commonly as the "metric system". While United States adoption of SI now appears imminent, the NACOME exploratory survey of elementary school teachers indicated that schools are not yet devoting much attention to the metric system -- apparently because available curriculum materials do not treat the topic adequately.

The SI implementation involves two changes in curriculum. First is use of a new set of base units, meters/kilograms/liters instead of

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* The remarks on calculators draw heavily on the paper by John Kelley and Ira Lansing prepared for the Indiana University project on "Problem Solving Strategies and the Use of Hand Calculators in the Elementary Schools". [5]

feet/ pounds/ quarts. But more important for mathematics teaching is a new system of derived units which are related by multiples of ten easily expressed in decimal numeration.

An NCTM Metric Implementation Committee has carefully assessed the impact of these changes and made recommendations on desirable practice in several key areas. They suggest that the adoption of new base units can be most smoothly accomplished by the strategy "think metric" that implies a variety of early school experiences to develop skill in estimating metric measurements and a minimum amount of conversion between metric and traditional English system measurements. The shift to decimal representation of measurements suggests a striking change in the importance of traditional arithmetic skill with common fractions. As mentioned in the preceding section, decimal arithmetic is generally fairly easy for elementary students since it builds on algorithms for whole number operations. On the other hand, common fraction operations require new skills that are traditional stumbling blocks for students through junior high school.

We urge that all school systems give serious attention to implementation of the metric system in measurement instruction and that they re-examine the current instructional sequence in fractions and decimals to fit the new priorities. The NCTM Metric Committee has produced helpful metric competency goals for students of various ages, and it appears that commercial publishers are preparing basal text and supplementary materials that will greatly facilitate metric instruction. In fact, the move to metric might well give a fresh boost to teaching about measurement, particularly emphasizing the importance of physical experience, estimation, and laboratory style investigations.

Statistics and Probability., Statistics, the science concerned with collection, analysis, and interpretation of numerical information

is important in the life of every citizen. It is needed for the proper evaluation of everyday matters such as advertising claims about gasoline mileage and relief from indigestion, public opinion polls and weather reports. It is indispensable for the solution of policy questions, from local affairs such as property assessment and predictions of school enrollments to national problems involving unemployment, crime, airplane safety and health. Even though numerical information is encountered everywhere, in newspapers and in magazines, on radio and on television, few people have the training to accept such information critically and use it effectively.

Many recent curriculum planning conferences and resultant development projects have given prominent attention to statistics and probability throughout elementary and secondary mathematics programs. While probability instruction seems to have made some progress, statistics instruction has yet to get off the ground. At the elementary school level, most common topics are only traditional graphing exercises and elementary descriptive statistics. Furthermore, the NCTM exploratory survey indicates that these topics get very little time in the average teacher's mathematics instruction. At the high school level probability topics in Algebra I and II texts are commonly omitted. A one semester senior course in probability and statistics has gained only a small audience of the very best students. Furthermore, this course places a heavy emphasis on probability theory, with statistics, if treated at all, viewed as merely an application of that theory. Though National Assessment gives reasonable attention to probability and statistics objectives, current commercial standardized tests do virtually nothing with these topics.

Realizing the need for broad based efforts if statistics is to take its place in the school curriculum, The American Statistical Association (ASA) and the National Council of Teachers of Mathematics (NCTM) in 1967 agreed to sponsor the Joint Committee on the Curriculum in Statistics and Probability. The committee decided to

focus its efforts on two tasks. First, school boards, principals, teachers, and parents would have to be persuaded that statistics is indeed important and useful and should become part of the elementary and secondary school curriculum. Second, materials would have to be prepared which teachers, willing to introduce statistical units in their courses, could utilize and consult.

To accomplish the first task, the committee prepared a volume of essays, Statistics: A Guide to the Unknown, in which well-known statisticians described in nontechnical language important applications of statistics and probability. A brief glance at the table of contents shows the broad spectrum of statistical applications: Man in his biological world; man in his political world; man in his social world; man in his physical world. It is difficult to think of human activities where statistics does not get involved in one way or other.

In considering its second task, the Committee saw no shortage of good probability texts and they had little doubt that secondary school mathematics teachers are able to offer work in probability to students who have the necessary mathematical preparation and who have the inclination to do the work. Statistics, on the other hand, presents quite a different picture. In spite of the rather widespread belief that statistics is a branch of applied mathematics, statistics has important extra-mathematical features to be presented along with the mathematical methods. In contrast to the case of probability proper, interesting and authoritative teaching materials in statistics at the pre-college level are hard to come by. It seems inevitable that for many years to come mathematics teachers will have to take over the bulk of secondary statistics instruction. But it is unrealistic to expect these teachers to assemble appropriate teaching materials on their own. The Joint Committee set out to do something about this void.

In four booklets entitled Statistics by Example, 52 real life problems with real data are explored in great detail, extending all the way from data organization to sophisticated model building. Each problem represents a series of mini-learing experiences or case histories, and each problem proposes additional exercises and projects. Some of the examples fit well in courses in the social sciences, biology, economics, civics, and even history and English, and reinforce the need to think of statistics not primarily in terms of its mathematical contents but as an interdisciplinary subject.

This past year and continuing into 1976 the Joint Committee has conducted working sessions at regional NCTM meeting to acquaint teachers with some of the materials. But much more needs to be done. Sales of the introductory book, Statistics a Guide to the Unknown, have now reached about 50,000 copies; however, it appears that the sales are mainly to college level students of statistics as a supplementary readings book, not to the intended lay public. Sales of Statistics by Example have reached about 15,000 copies, but again teachers who have tried the materials indicate need for more elementary resource material and specific guidelines on where and how to use statistical topics in the normal curriculum. Development of these materials should have high priority in curriculum making. But most important is acceptance of the premise that statistics can and should be taught all the way from Kindergarten through grade 12.

NACOME recommends that instructional units dealing with statistical ideas be fitted throughout the elementary and secondary school curriculum. The following are some possible ways to do so.

1. Use statistical topics to illustrate and motivate mathematics.
2. Emphasize statistics as an interdisciplinary subject with applications in the natural, physical and social sciences and the humanities. Possible interdisciplinary courses: a course oriented towards computers and statistics; courses in the physical, biological and social sciences using statistical tools.

3. Develop several separate courses dealing with statistics to meet varied local conditions. Two possible courses are:

   a. A statistics course for high school students with little or no algebra, especially non-college bound students or college bound students in the social sciences who as consumers and citizens must learn to cope with numerical information. The main theme of such a course would be "making sense out of numbers" without getting involved with complicated mathematical formulae.

   b. A senior year statistics course with a probability prerequisite for more mathematical and scientifically minded students. The need for such a course would be greatly enhanced if CEEB introduced an Advanced placement Program in Statistics and by the inclusion of substantial statistical problems on standardized tests.

Summary. The above outline of prospective curriculum development in school mathematics suggests an exciting period of innovation ahead. Better use of applications, calculators and computers, or probability and statistics, and appropriate response to career education demands will necessitate substantial curriculum development and teacher education efforts. Impending adoption of the metric system of measurement will require a similar national effort K-12. To meet this challenge, it is imperative that teachers, teacher educators, and research scientists from pure and applied mathematics reestablish the powerful cooperative working relationships that characterized innovative efforts throughout the 1960's.

2.2 Curricular Impact of Pedagogical and Accountability Trends

Curriculum content innovation potentially or already available to school mathematics could produce during the next decade an even greater change than the "revolution" of the 1960's. But the major issues in curriculum change may well be generated by forces directed at changing instructional and evaluation practice rather than content.

The most fundamental of these forces is the increasing movement toward specifying goals of education as precise performance abilities to be acquired by students. Grounded in a combination of behaviorist psychological theory and public demand to make educational institutions more clearly accountable for their efforts, the behavioral objectives style of curriculum planning and evaluation has become common at every level of education. Because mathematics education is perceived to have readily described and measured skill objectives, the school mathematics curriculum is facing the challenge to specify its objectives.

As part of the NACOME effort to survey the status of mathematics education in the United States, we have collected whatever published mathematics objectives were available in each of the 50 states.* The diversity of this collection defies detailed quantitative analysis, but there are some general patterns and important implications suggested by the survey.

With few exceptions, state departments of education and state supervisors of mathematics have traditionally played an advisory and resource role in curriculum and instructional development. This role seems to be changing -- with over 30 states now reporting some form of mathematics goals or objectives, most of them developed recently. In 12 states the objectives were developed in response to legislative accountability mandates; in another 20 states the initiative came from state department of education concern -- quite often as part of "needs assessment" efforts required to obtain federal support for educational programs under Title III. In 15 states the mathematics objectives are clearly related to regular assessment programs; that is, the tests are constructed to match stated objectives or the objectives are constructed by analysis of a chosen assessment test.

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*We must acknowledge the important assistance of the Association of State Supervisors of Mathematics in conducting this survey.

The objectives themselves vary widely in organizing framework, specificity, grade level focus, and content. For instance, in Pennsylvania one of Ten Goals of Quality Education states simply:

> Quality Education should help every child acquire to the fullest extent possible for him mastery of the basic skills in the use of words and numbers. [6]

The Michigan Minimal Performance Objectives for Mathematics are arranged in a detailed hierarchy for grades K-9 with objectives like:

> Given a set of labeled fractional cut-out parts including several unit wholes, the learner will demonstrate the result of adding two mixed numbers with like denominators of 2,3,4,6, or 8 by fitting the appropriate parts together and writing the sum as a whole number and a proper fraction. [7]

Each objective is accompanied by sample assessment items.

The range of objective styles between these extremes includes moderately specific performance objectives, topic lists organized by grade or by traditional high school courses title, scope and sequence charts arranged into content strands, or (in a few cases) only specified levels of attainment on standardized tests.

For instance, typical Wisconsin objectives for grade 4 expect that students should be able to

> Determine the factors of a counting number.
> Read and write numerals as needed.
> Recognize parallel lines as lines in a plane which do not intersect. [8]

Florida establishes, each year, collections of priority objectives to serve as the basis for state assessment. These objectives are not intended as a comprehensive curriculum. Rather, they "idenify skills (1) which educators throughout the state consider important for students to learn, and (2) which can be measured in large group, standardized testing situations." Some samples from the 1973-74 objectives for grade 3 include

> Given addition exercises with three or more addends which are grouped within parentheses, the learner names sums through 18.

> Given multiplication expressions involving products less than 30, the learner selects the picture which represents each expression. [9]

The California objective scheme is typical of the content strand by grade level organization. The main content strands for grades K-8 are: Applications and Problem Solving, Arithmetic/Numbers/ Operations, Geometry, Measurement, Probability and Statistics, Relations and Functions, Logical Thinking. But the presentation of objectives within this framwork is not a list of discrete behavioral expectations. It is an expository guide to instructional approach and emphasis. For instance, from the content strand on measurement:

> Arbitrary Units of Measure
>
> In the introductory stage, pupils first become familiar with the properties of the objects to be measured. Next they learn to make discriminations among those properties. They then learn to compare objects according to the quantitative properties they possess in terms such as "is equal to", "is less than", or "is greater than"... [10]

Kentucky objectives are apparently dictated by the state assessment program. For instance, one 1973 objective was

> During the spring semester of the eighth grade, Kentucky pupils will demonstrate application of arithmetic computation by attaining an average grade equivalent score equal to or exceeding the criterion (8.7) as measured by the Arithmetic Computation Subtest of the Comprehensive Tests of Basic Skills, Form Q, Level 3. [11]

It seems likely that the assessment test plays a substantial de facto role in setting goals for mathematics instruction in many other states as well.

While NACOME was unable to make a similar comprehensive survey of objective setting practices in local systems and individual schools, signs point to equally extensive and diverse activity at those levels. The NCTM exploratory survey of second and fifth grade teachers showed 76% of the districts have published mathematics objectives for students. Of those who reported objectives in their state or district,

52% claimed to base their teaching on those objectives regularly and another 20% sometimes for some topics.

The process of generating state, system, and school mathematics objectives has clearly absorbed a great deal of professional time and energy. This allocation of resources and the educational consequences of adopting behaviorist principles in curriculum and instruction have not gone unchallenged. So it seems appropriate to ask "What are the potential payoffs from constructing state or local system objectives?" and "What forms of objectives are most likely to achieve the varied goals?"

Arguments in favor of constructing precise behavioral objectives for mathematics instruction promise benefits in a wide range of instructional activities. Some argue that objectives facilitate design and writing of text material, planning for classroom teaching, and construction of fair and comprehensive evaluation instruments. Others cite the usefulness of objectives in setting minimal competence goals for all students and making schools accountable for achieving their educational tasks. Several states have begun implementation of "learner verification" laws that require publishers to provide evidence that their material is effective in achieving its stated objectives, prior to approval of that material for purchase under the state text adoption program.

A more fundamental argument for the behaviorist point of view is the contention that learning a mathematical principle, concept, or skill is dependent on acquiring a collection of simpler component performance abilities organized into hierarchies of learning dependence. This hypothesis suggests analysis of mathematics learning tasks into subordinate behavioral objectives, design of instruction to accomplish each of these components, and individually sequenced and paced instruction that proceeds from pre-test (for placement in the learning hierarchy), to instruction, to post-test, with the goal

mastery of each objective. Attempts to implement this type of "individualized" instruction have led to widespread localized curriculum development in which systems, departments, or individual teachers write their own objectives and key those objectives to learning assignments in a variety of different textbook resources.

Opponents of behaviorism counter that objectives of a mathematics curriculum are really implicit in the problems set for students in each section of a text. They claim that detailed specification of behavioral objectives falls far short of creating lively instructional material or classroom activity, and focuses on trivial, low-level skills because those are easy to translate into the limited language of behaviorism. They argue that rule by objectives in school mathematics discourages open-ended problems that stimulate student creativity, offering instead a collection of disjointed training exercises that form a ceiling (not a foundation) for the mathematical vision and ambitions of students.

The objection to behaviorism in mathematics education also reflects deep disagreement on fundamental psychological issues of learning theory. The cognitivists claim that while learning of a mathematical principle or skill might occur by acquisition of many simpler related skills, this is not in any way evidence that the optimal learning situation presents those skills in serial order. They argue for the importance of giving students generous experience in unravelling complex problem situations, acquiring in the process many heuristic skills that generalize far beyond any specific situation.

One special manifestation of the current interest in objectives and accountability is widespread concern for establishing minimal mathematical skill lists. Responding to the question of what mathematics is required by every citizen in order to be an intelligent consumer, an employable worker, or an informed participant in social/political decision making, various state departments of education, an

NCTM Committee, and many individuals have generated basic competence recommendations.

NACOME proposes no list of our own or endorsement of any other list. However, we feel it is important to analyze the effect that this quest for minimal mathematical requirements can have on total program planning. Identifying specific minimal goals for mathematics instruction can help assure that important objectives are not overlooked and that reasonable instructional effort is devoted to those objectives. At least this assumption is inherent in the widespread support for state and local objective and assessment schemes. On the other hand, focus on minimal objectives has several potentially serious drawbacks.

First, the objectives intended as a minimum for mathematical achievement can all too easily become a ceiling also. Particularly when the objectives are tied to an accountability assessment, there is a reasonable tendency to set very modest goals. Second, focus on minimal skill goals can inappropriately constrain planning for instruction by suggesting that skills must be acquired in a rigid sequence of mastery level steps. Though arithmetic computation is important for problem solving, one need not achieve complete mastery of arithmetic before encountering meaningful problems. Some concept areas, not commonly represented in minimal learning goals, serve as excellent vehicles for teaching basic skills. For instance, valuable practice with arithmetic and graphing skills can develop in the course of a unit on statistics and probability. Third, in a cumulatively structured subject like mathematics, it seems highly unlikely that fundamental concepts and skills will be passed over by instruction reaching for higher goals.

Any curriculum design is based on a prediction of the information and skills that will be most useful to young people many years after the time of instruction. In the rapidly changing technical

world we experience today, it is hard to have much confidence in any very specific list of essential skills. If the task of identifying minimal mathematical competencies leads to careful analysis of the fundamental concepts and methods that have characterized the discipline throughout its history, it can provide useful insight for curriculum development and teaching. If, as we fear, the search is for a list of easily taught and easily measured skills it will not be productive.

One of the unfortunate patterns in American response to educational innovation is a tendency to draw hard and fast battle lines between dichotomous positions. From both sides the debate over behavioral objectives and their uncritical application in mathematics instruction has taken on an "all or none" tone. The members of NACOME believe that the extreme and hard positions taken by some persons on each side of the debate are unhealthy and that both points of view lend valuable insight to the design, execution, and assessment of mathematics instruction. But we agree that implementation of an extremely narrow, overly-specific conception of behavioral objectives is a severe danger to mathematical education and we deplore much of what is current practice in producing more and more lists of low-level, narrow skills as the sole or major objectives of mathematical instruction. We challenge the uncritical acceptance of the engineering or management metaphors as the best models for educational practice.

We urge all mathematics educators to tackle the task of developing and measuring attainment of the attitudes, problem solving, and critical thinking abilities traditionally judged so important in mathematics teaching. We urge similar experimentation with flexible curriculum structures that nonetheless convey to students the powerful unity of mathematical ideas and methods. And we urge that these mathematical ideas be offered in a variety of instructional modes until or unless sound research demonstrates (far more convincingly than evidence now available) superiority of particular methods.

CHAPTER 3. PATTERNS OF INSTRUCTION

Public controversy over the quality of school mathematics has tended to focus on issues of content, sequence, and priority in the curriculum. However, much of the urgent debate within the profession has centered around recent innovations in teaching styles and organizational patterns.

The reader is urged to read this chapter with caution, avoiding over-generalizing. We discuss trends and new directions and changing patterns as they are seen in the more visible programs, for they lead educational reform. But we are brought up short by surveys suggesting that very little of this is in evidence in many classrooms across the country. Space devoted to new directions is in no way commensurate with magnitude of present impact.

At the secondary school level, proponents of "new math" commonly espoused instruction in a style of questioning which would lead students to discovery of mathematical ideas. Implicit in this advocacy of the Socratic teaching model was belief that such learning would result in deeper understanding, better retention, and easier transfer to novel problem solving situations. At the elementary level recent findings of psychological research in development and learning have been translated into prescriptions of instructional style that alter significantly the traditional patterns of teacher/student interaction. Piagetian developmental theory has reaffirmed the importance of

readiness in planning for concept instruction, and emphasized the role that action on physical models of ideas plays in such learning. As a consequence, there has been a strong effort to base much elementary and junior high school instruction on laboratory or activity center models utilizing many forms of manipulative materials and patterns of child/child interaction in place of the traditional teacher-directed large class.

Sensitivity to readiness of learners quickly demonstrates the widely varied skills and aptitudes possessed by any group of children and makes a strong argument for increased individualization of instruction -- matching the content, presentation mode, and pace of teaching to the unique capabilities of each student. Current popular models of individualized instruction, based on a behaviorist model of learning, cast the teacher in a role of educational consultant. At all levels of instruction there are experiments in which alternative media act as agent and manager of instruction.

3.1 Alternatives in Classroom Instruction

A typical characteristic of proposed new mathematics programs at the elementary school level has been the development of alternatives to traditional teaching practices. Many of these challenges to the common format of teacher-directed question and answer development and written seat work ("supervised study") fit in one of three categories: the wide variety of "individualized" instruction procedures or classroom organizations, mathematics laboratory activities and the use of manipulative materials, and interdisciplinary or problem-centered courses.

Individualized Instruction. The basic components of an individualized program are: a set of objectives to be achieved, diagnostic pre-tests for each student for each unit of study, a prescription to guide students through the objectives to be attained, post-tests, and an opportunity to restudy to mastery. Ideally the student may

determine personal goals both in content and time, a preferred type of learning material and pace of learning.

Unfortunately, there are few, if any, ideal individualized programs, although devotees and users of various types argue their case forcefully. Critics of individualized schemes focus on four major weaknesses. First, few such programs really match instruction to learner styles. The only individualized feature is pace since all students cover essentially the same materials in the same manner. The predominant mode of instruction in these programs is teacher-prepared or commercial worksheets or programmed texts. Until recently, there have been few attempts in such programs to allow students the opportunity to develop concepts through use of concrete materials or to learn via any other mode of instruction. For example, one of the early individualized programs was the Individually Prescribed Instruction - Mathematics (IPI-Math) developed at the University of Pittsburgh. This program is built on a sequence of 359 instructional objectives with a student booklet for each behaviorally specified skill. Students take a pretest to determine their placement within a specific skill and the teacher then writes a prescription for the student from one of the skills booklets. Teaching is done by the booklets with little or no outside help for the student. Since each student is expected to pass through all the skills in the booklets, the "individualization" refers to the pace at which the student works. The basic characteristics of individualized programs embodied in IPI-Math are now found in numerous other experimental and commercial programs.

While evaluation of such programs indicates that students learn the mathematics skills included as objectives, critics maintain that many of the problem-solving skills and attitudes considered to be part of a good mathematics program are missing. Responding to this concern, a newer version of IPI makes use of manipulative materials as an adjunct to the regular skills booklets. For the most part,

however, the program emphasis has remained building computational skill.

The second main criticism is that most individualized programs deny the value of students discussing mathematical problems with each other and with a teacher. The critics maintain that most learning theories imply the importance of student interaction with other students, with teachers, and with their environment. Probably far too few of the individualized programs in widespread use have provided for such interaction. Considering the limited time devoted to mathematics in the instructional program, it is clear that in a narrowly defined "individualized" program the amount of time a teacher spends with each student is very small.

The third major criticism of individualized programs is that the emphasis on testing until mastery tends to lead students to shallow learning of "local" rules and to emphasis on low-level skills. Major concepts are often difficult to generalize solely from paper and pencil activity, particularly at the lower grade levels. The necessary abstractions often have not been based upon the concrete experiences of the children. Thus, there is a tendency by the child to learn just those rules which will allow passing the post-test. He or she may then be unable to integrate and apply the skills or concepts in novel situations.

Finally, a major difficulty with most diagnostic/prescriptive individualized programs is the quantity of required record-keeping. The teacher must keep track of what each student has accomplished, who needs to be tested, what prescriptions need to be made next. The time consumed by these tasks reduces the time for helping students. Some local school districts, as well as commercial publishers, are developing computer managment systems for individualized programs. These computer systems generate and administer diagnostic and post-tests; make individual prescriptions for learning, drawing on a

variety of teaching materials; and keep records of student progress. As these systems become cost effective, they may signal a true breakthrough in personalizing instruction for each student. Currently, less costly alternatives are also possible. With appropriate training and guidance, paraprofessionals, student clerks, or the students themselves can do the recordkeeping.

Individualization of mathematics instruction at the secondary school level has grown in popularity immensely during the last 5 years -- though data indicating the extent of such innovation is limited. There have been fewer curriculum development projects or commercially published systems designed for use in individualized algebra, geometry, or advanced mathematics courses. But secondary mathematics specialists are much better prepared to produce their own objectives, tests, and guides to learning materials for individualized systems.

At the secondary level modular scheduling has, at least in theory, provided a promising vehicle for individualization and independent study. For example, modular scheduling may provide for small and large group instruction as well as individual tutoring on specific problems, counseling on advanced independent work, and special research projects not otherwise available within the standard mathematics class pattern. Secondary schools are increasingly using media/learning centers as the logical outgrowths of the library, and satellite centers specifically designed for mathematics activities are encouraging the increased personalizing of mathematics learning.

Activity, Multimedia, Laboratory, and Materials-Oriented Programs. There has been a great deal of published enthusiasm for mathematics laboratories and the use of manipulative materials, K-12. Perhaps the most recent historical antecedents of this category of organizational style have been, along with other sources of influence, the Madison Project in the United States and the Nuffield Project in England.

The Madison Project was designed to develop a system of teaching activities based on discovery approaches to learning. The project lessons and materials emphasize the importance of doing mathematics rather than simply learning it; and, as a consequence, they stressed stories, games, and concrete materials to learn new concepts. The approach required interactions of students with materials and students with students. While the Project was never meant to provide a complete mathematics program, it influenced and was influenced by others which did provide, in a similar conceptual framework, a more complete mathematics program. The Nuffield Mathematics Project and the Mathematics for Schools programs in England met this challenge. These programs were based on the concept of active learning, children doing mathematics, seeing mathematical implications in everyday situations, and conducting mathematical investigations using everyday materials and ideas. Children worked in teams or individually on topics that piqued their interest and which had been previously developed through task card activities.

In addition to the availability of varied manipulative materials, increasingly popular open space programs are generally characterized by the use of varying instructional styles, multimedia approaches, variable-sized groupings, and learning (interest) centers. After an initial motivating lesson by the teacher, students individually or in small groups select a center and begin working there at the task indicated. These tasks ideally consist of problems or leading questions designed to require the student to experiment, infer generalizations, draw conclusions, and predict other consequences based on the generalizations. In ideal form, opportunities for review of the mathematics concepts or skills needed to work on the problem are provided within the framework of the task or as a prelude to the task. Often students rotate through several centers during the development of a particular unit. Goals of the use of learning centers may include development of basic skills or the extension of these skills into higher order mental processes or concept building.

Criticism of laboratory or activity center patterns of instruction focuses on the degree of structure and systematic objective development involved. Outcomes are not always precisely stated or tested and clear-cut, systematic goals may not be evident to the observer. It is often contended that learning outcomes are fragmented, uneven, difficult to assess or even based only on "faith". Involvement, motivation, and intense activity do not necessarily guarantee significant learning.

Several current experimental programs are attempting to meet the common criticisms of individualized and laboratory style instruction with a blend of activity methods, varied concrete materials, student interaction with peers and the environment, and carefully planned objectives and evaluation of student progress. The Developing Mathematical Processes (DMP) program at the University of Wisconsin attempts to meet those goals. The Comprehensive School Mathematics Project (CSMP) in St. Louis is another promising form of individualized program -- very different from those with a managerial orientation.

Even if teachers or school systems are unable or unwilling to implement an activity based total mathematics program, it is possible to enrich regular instruction by using some of the manipulative materials that embody abstract mathematical ideas. One major longitudinal study has pinpointed the effectiveness of such teaching by teachers who had, themselves, gone through an inservice training program directly related to the use of manipulative materials. The Specialized Teacher Project [1], one of the California Miller Mathematics Improvement Programs, demonstrated over a three-year period that students at all grade levels achieved better and had improved attitudes when taught by teachers who had specific training in the use of mathematics laboratories and manipulative materials. However, in spite of the recent publicity and emphasis it is not at all clear that manipulative materials are widely used. For instance, 37 percent

of the elementary school teachers in the NCTM survey had never used the mathematics laboratory, and ten percent had never used manipulative materials at all.

Problem-Centered and Interdisciplinary Programs. Perhaps the instructional pattern approached with most reluctance by mathematics teachers is the interdisciplinary program which integrates mathematics with the sciences and other subject areas. Many such programs have been attempted during the past 15 years, and up to now we have seen little lasting effect on the mathematics curriculum. An earlier discussion in Chapter 2 outlines the aims of MINNEMAST and USMES as examples of programs in this category.

There are also current projects experimenting with problem-centered programs in which the emphasis is on mathematics. In one, located in centers at the Indiana University, the University of Northern Iowa and schools of Oakland County, Michigan, a prominent component is work with calculators.

The interdisciplinary program concept is achieving a considerable degree of implementation in association with the middle school movement. The middle school is a different administrative section of the grade level pattern, usually (not strictly by definition, though) comprising those levels for children about 10-13 years old. But many people consider middle schools a movement with differing emphases, identity and definition. Many consider an essential characteristic of the middle school to be its emphasis on the interrelatedness of subject areas rather than their distinctions.

Some critics urge caution in this approach, expressing fears that in such programs mathematics becomes overly dominated by the sciences and other areas and thus not only subsumed but gradually squeezed out. They also object that there are justifications for teaching mathematics as a human endeavor in its own right, not

solely as a support of or application to the problems and methods of other fields.

Summary. Most exemplars of the instructional patterns discussed above are to be found at the level of elementary, middle, or sometimes junior high schools. It appears that less change in instructional methodology has occurred in the high school, although the influence of mathematics education reform can be seen in a tendency toward more Socratic dialogue or genuine discussion rather than the "lecture and problem set" tradition.

With an increasing array of patterns of instruction being developed and advocated, a sound empirical base for choice and selection is needed by teachers and administrators. Unfortunately, such a basis for decision-making is not yet available. Research does not present a useful answer at present. It is not that there has been a lack of investigation of this important question. There have been many studies attempting to determine the effects of various methods of instruction. But they do not present strong and conclusive evidence and almost all suffer from weaknesses in design. For the most part, the particular methods being studied are poorly and vaguely defined. Most are based on a small sample and often lack a convincing methodology to assure that the method purportedly being used actually described what happened in the classroom. The large projects experimenting with various methods of instruction have done little summative evaluation. It seems fair to say then that research presents no convincing case for a particular methodology or pattern of instruction at this stage.

3.2 Alternatives in Instructional Staffing

Alternatives in classroom patterns have been accompanied by related variations in the role of teacher and other instructional personnel. Examples are team teaching, differentiated staffing, and the use of paraprofessionals or volunteer help.

There has been little research relating variables of instructional staffing and patterns of curricular organization. A study by Conant [2], to be discussed later in this chapter, suggests that in elementary school the percentage of mathematics class time devoted to individualization increases when paraprofessionals are used. In that study, while the total time allotted to mathematics remained at about 18 minutes per day, the time in individual work increased by more than 40 percent.

In the NCTM survey, 68 percent of the teachers indicated that they had not participated in any kind of team teaching. In fact, 40 percent of the respondents indicated that they taught whole-class instruction 50 percent or more of the time. One type of teaming, the use of specialized teachers, does appear to be prevalent, however, particularly at the upper grade level in elementary schools. Of the fifth-grade teacher respondents to the NCTM survey, 38 percent indicated they were specialized in mathematics. However, there appeared to be very little difference in the methods that they used as compared to the traditional self-contained classroom teacher. [3]

It is apparent that we need a body of research bearing on the complex interrelationships among classroom organization and variations in instructional roles and staffing characteristics.

3.3 Alternatives in Media and Materials

Presumably, changing methods and instructional patterns imply changes in utilization of different media and materials. Individualization usually entails a greater reliance on manipulative materials; multiple texts; tapes, cassettes, films and filmstrips; or perhaps technology-based media such as the computer and instructional television. But aside from manipulative materials it does not appear that any of these other media is yet making large inroads in the mathematics classroom. In fact, the NCTM survey showed that 56 percent of the teachers used a single textbook. Further, the study

has indicated that 78 percent of the teachers have never used instructional television, 83 percent have never used hand calculators, and 81 percent have never used computer-assisted or computer-mediated instruction.

This is complicated by the fact that studies investigating the relative effectiveness of various media of instruction have revealed no consistent, significant pattern of results favoring either technology-based or teacher-centered procedures. In 56 studies of instructional television compared to traditional instruction in mathematics, 89 percent showed instructional television doing as well as or better than traditional instruction, particularly at the lower grade levels. [4] Although fewer studies have been done for computers, similar results have been indicated.

While development in the area of television and computers is very active, these media have not yet been fully exploited as teaching tools of mathematics. It appears that while there is some promise, this promise has not been realized in most of the mathematics classrooms in the country.

Both television and computer-mediated instruction meet with strong opposition from many teachers. Part of the opposition may come because the early advocates "came on too strong" and oversold. Part of this opposition also may come from the threat teachers feel because they do not know how these new tools will change their roles or even how the tools may be used effectively. Further, technological advances in the fields are still unknown to most teachers. Early television programs, for example, were little more than talking faces presenting materials in ways many teachers felt capable of delivering themselves. Recent advances in television communication, however, have led to Sesame Street-type programs which allow for viewer participation or response and immediate follow-up by the classroom teacher.

Computers, too, have moved beyond the early drill and practice systems. Mathematics students are now able to use computers and programmable calculators as problem-solving tools, as well as in student/machine interaction, at a level beyond simple drill and practice. Individual computer companies have developed user group publications which have allowed schools to exchange programs and cooperate in curriculum development activities. A major problem with computer-mediated instruction is cost effectiveness. While costs for some large districts may be low because of volume and the ability to tie into existing facilities, the computer is still out of the cost range for most smaller school districts. It is likely that a lower price tag would result in much wider use of the computer in mathematics classrooms. Breakthroughs in the microcomputer field may soon provide major developments here.

3.4 Research on the Teacher and Classroom Instruction

When any report, conference or survey attempts to present a broad view of what is happening in education, it is unavoidably but justifiably subject to the criticism that it presents a distorted view. One of the frustrations of working in the educational enterprise is that the very magnitude of the endeavor prevents our ever getting a truly accurate picture of what is really happening in ordinary classrooms across the entire country. When reports and conferences discuss what is "current", meaning what is going on at the cutting edge of innovation, they sometimes leave the impression that it is a time of great change and ferment. Meanwhile, back in the ordinary classroom -- the classroom which is not a part of some well-publicized, well-funded project -- we may find little evidence of profound differences over a decade or so.

The overwhelming feature of the educational system is its conservatism, inertia, and imperviousness to sweeping, profound change. It accepts, accommodates and swallows up all sorts of curricular fashions and practices. This is not to say there is never change.

But reformers may have to be content with the gradual evolution characteristic of The School, as the product of many, many small but important increments rather than radical revolution. Perhaps the multi-faceted, complex, bewildering array of alternatives being tried today represents a recognition of this fact rather than the confused state or lack of focus which some educators despair. Indeed this is a lesson we may have learned from "new math" which was, after all, a heady time of common purpose and singular focus aimed at sweeping change.

The alternative instructional patterns discussed above are trends in the forefront of educational change. They are the highly visible, well-publicized manifestations of the educational system's need to experiment, change, adjust, reform. But they are not necessarily descriptive or even suggestive of what goes on in the majority of classrooms.

The question "What goes on in the ordinary classroom in the United States?" is surely an important one, but in attempting to survey the status of mathematical education at "benchmark 1975," one is immediately confronted by the fact that a major gap in existing data occurs here. Appallingly little is known about teaching in any large fraction of U.S. classrooms. Such a situation is intolerable in the face of the growing emphasis on assessment, since exhibited effect generates a need to find cause, and much of today's controversy centers on assumed and suspected causes for reported test results. The vacuum of data on classroom practices should should give pause to those who present simplified cause-and-effect explanations. We need information, not scapegoats.

The NCTM Survey. In an effort to get some information on trends in teaching, NACOME undertook an exploratory survey of the characteristics and teaching practices of elementary school teachers. The survey, which has been referred to earlier in this report, was

funded by the National Council of Teachers of Mathematics and conducted by Jack Price, Jonathan Kelley, and J. L. Kelley. The detailed description of results which follows is taken directly from their final report to the NCTM. The findings represent responses from 1,220 teachers (over 40% return) from a wide variety of classroom situations across the country.

a) Procedure of the Study. The study was conducted through use of an anonymous questionnaire. Three hundred supervisors from a list of more than 800 provided by the NCTM were randomly selected and asked to distribute ten questionnaires each. These questionnaires were also to be distributed randomly according to detailed procedures provided to the supervisors, five to second-grade teachers and five to fifth-grade teachers in the area served by each supervisor. A post card was provided each supervisor to indicate when all the questionnaires had been distributed. Post cards were received from 191 supervisors indicating at least 1,910 questionnaires had been distributed.

It is conceivable that the method of selecting respondents through supervisors introduced a bias in the results, since not all districts have mathematics supervisors. It might be assumed that such a bias would result in the group of respondents being more professional, better prepared, and more likely to try innovative methods in mathematics teaching than the average teacher.

b) Characteristics of Teachers. The characteristics of the teachers are very important, since the remaining findings should be considered in light of these characteristics. Differences between respondents by geographical areas, type of districts, socio-economic status of the school, and grade taught are small. Fifty percent of the respondents were first-grade teachers, 47 percent were second-grade teachers, and three percent declined to answer the question. They were relatively young -- 53 percent had been teaching ten years

or less, and 56 percent were 40 years of age or younger. Over 90 percent of the second-grade teachers and 70 percent of the fifth-grade teachers were women.

As a group, the teachers liked mathematics -- 65 percent found the teaching of mathematics very interesting, 45 percent preferred teaching mathematics to reading or social studies, and only 11 percent liked it least of the three. Also, 53 percent thought it was the easiest of the three subjects to teach successfully.* As further evidence, 53 percent believed that their present students were doing better than past comparable classes, and only 20 percent thought their students were doing less well than similar classes.

Eighty-eight percent of the teachers had had two or more semesters of high school algebra, 70 percent had at least two semesters of high school geometry, 63 percent had at least two mathematics courses in college, and 48 percent had at least two mathematics education courses in college.

Two of the most startling statistics are that 84 percent do not belong to any professional mathematics organization, and yet 38 percent of the fifth-grade respondents are specialized teachers of mathematics. There was no dramatic difference between the specialized teachers and the others in either training or point of view. The specialized teachers had a little more mathematical background and were somewhat more favorably disposed toward mathematics teaching.

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*In a study by Jonathan Kelley in the Berkeley, California school district only 22 percent of the teachers preferred teaching mathematics to teaching the other two subjects, and only 20 percent reported that their students were more interested in mathematics than the other two subjects. But, 43 percent of the teachers in this survey reported that their students were most interested in mathematics. [5]

The picture of the elementary school mathematics teacher which emerges is generally encouraging. The findings of the following sections consequently take on increased meaning as they relate to the implementation of new programs and new methods of teaching in the elementary mathematics classrooms.

c) Objectives and Assessment. The questionnaire was designed to elicit data regarding objectives and assessment, textbook topics and usage, time in specific classroom activities, and teaching methods and procedures.

A major effort in mathematics education, as in many other subject matter areas over the past few years, has been the development of instructional objectives. Most states now have some kind of objectives at least for mathematics and reading. Further, many states and most large districts have developed assessment plans of varying complexity and value. It was the purpose of one set of questions to explore whether the extensive work had, in fact, made an impact on the classroom.

Apparently, this word has made its way to the classroom. Eighty-three percent of the respondents reported that either the state or the local district or both had published objectives for mathematics. Only 13 percent did not know about objectives, and three percent indicated that neither the state nor the local district had them. But, of those who indicated knowledge of objectives, only 63 percent said they made a conscious effort to use them in their teaching.

Further, 77 percent indicated that either or both the state and the local district had some sort of mathematics assessment. Another 11 percent did not know and ten percent indicated that neither their district nor the state had mathematics assessment. Of those who reported mathematics assessment, 43 percent said they based their teaching on the results of the assessment.

d) Textbooks and Topics. A major recommendation of the mathematics reform movement has been a decreasing reliance upon a single source of mathematics information coupled with an increase in topics not previously covered in elementary texts. Some state or district "textbook" adoptions have become "instructional systems" adoptions in which a multitude of supplementary and audio-visual materials and equipment are made available. Among topics considered new at the elementary level ten years ago are probability, statistics, the metric system, and relations and functions. One set of questions was designed to determine whether new topics were available and taught and whether multiple materials were used in this teaching.

Of the respondents 56 percent used a single textbook. Another 26 percent used one predominantly among two or more. Only seven percent used no basic text. Interestingly enough, 65 percent of the second-grade teachers used a single text, but only 49 percent of the fifth-grade teachers did. Their texts were relatively new and fairly satisfactory; 60 percent reported a copyright date of 1970 or later and 70 percent were very or fairly well-satisfied with the book. Texts that emphasized skills over concepts were preferred by 42 percent, and another 48 percent preferred texts that emphasized them equally. Only 2 percent preferred texts with emphasis on concepts over skills.

The texts were followed closely by 53 percent of the teachers. However, more than half of the teachers reported that their students actually read less than one page, or at most one or two pages of textual materials out of every five. It seems likely that texts are used primarily as a source of problems.

Many questions asked teachers to report the topics stressed in their classes. Detailed discussion of these findings appeared in Chapter 1. We only recall here that many of the "new math" innovations like geometry, statistics, probability, relations, functions,

and the metric system have apparently made little headway against the traditional domination by arithmetic computation.

e) Class Time. Earlier studies have estimated actual time spent during a school day in mathematics activities as ranging from 18 to 37 minutes. The literature reports a trend toward increased time spent on mathematics in the elementary classroom. This study appears to support the contention.

The average student is exposed to mathematics one-half hour or more in ninety percent of the classrooms studied. The weighted average time in mathematics class for all respondents was 43 minutes.* This large time span could have been attributable to the number of mathematics specialists at fifth-grade level. However, as it turned out, 40 percent of the self-contained classroom fifth-grade teachers spent 50 minutes or more daily on mathematics while only 31 percent of the specialist teachers spent that much time. A further breakdown by grade level shows that 83 percent of the fifth-grade teachers spend 40 minutes or more on mathematics daily, while 55 percent of the second-grade teachers spend that much time. On the other hand, only five percent of the fifth-grade teachers reported less than 30 minutes, compared to 14 percent of the second-grade teachers.

Of the 43-minute average class time, 43 percent was spent in written or seat work. Thirty-six percent was spent by teachers discussing or explaining and 21 percent in other activities. Thus, seat work occupied 20 minutes or more for 86 percent of the respondents. Less than 20 minutes was spent discussing and explaining by 51 percent of the teachers, and 65 percent spent less than 15 minutes on other activities.

*Teachers in the Berkeley study also claimed 44 minutes per day of mathematics teaching. However, they in fact spent above 37 minutes, as indicated by a question of the type, "Of course today may not have been typical, but how many minutes did you spend on mathematics today?" [6]

f) Teaching Methods. The new mathematics movement has been characterized in part by changes in the instructional patterns in the classroom. A great deal of attention has been given to Socratic or discovery approaches. Laboratory experiences and reliance upon concrete, manipulative materials have been the mainstays of many new instructional modes. Diagnostic/prescriptive teaching, individualized and small-group instruction, team teaching, and learning centers have been widely reported as activities of modern classrooms. New technology in the form of computer-assisted and computer-mediated instruction, instructional television, and hand and desk calculators have also been indicated as having impact.

At least for the teachers in this sample, there appears to be more telling than doing with respect to the above instructional innovations. Forty percent of the respondents use whole-class instruction most of the time, but only six percent say they have never tried individualizing. Forty-three percent of the second-grade teachers use whole-classroom instruction more than 50 percent of the time, and 39 percent of the fifth-grade teachers use this method. (This is not entirely consistent with their belief as expressed later in the questionnaire. Only seven percent felt that mathematics should be taught in a single group as compared to 84 percent who favored grouping by ability.) It is interesting to note, however, that the specialized teachers at both levels use whole-classroom instruction significantly less than the self-contained classroom teachers at both levels. For example, while 46 percent of the second-grade self-contained teachers use whole-class instruction 50 or more percent of the time, only 28 percent of the specialized second-grade teachers do.

As might be expected, the second-grade teachers use manipulative materials significantly more than the fifth-grade teachers; 21 percent of the second-grade specialized teachers indicate their use nearly every lesson. As a whole, 72 percent of

the respondents say they use some sort of laboratory experience only occasionally (less than ten percent of the time) or never. Computer-assisted or computer-mediated instruction has never been used by 81 percent of the respondents, and 78 percent have never used instructional television. Eighty-three percent have never used hand calculators, and 68 percent have never tried any type of team teaching.

g) Teaching Experience. In various parts of this report, characteristics of a "modern" approach to teaching mathematics have included use of multiple text materials, individualization of instruction including diagnostic/prescriptive teaching, and the use of manipulative materials, particularly at the early grade levels. The survey found that the use of these did not vary much according to teaching experience. The following table breaks down the three processes by grade level percent of responses:

Table 3.1

Percent of Respondents Indicating Each Procedure According to Teaching Experience (Years)

	Years of Teaching Experience				
	1-3	4-10	11-15	16-25	26+
GRADE TWO					
Single text	64	69	62	60	66
Whole class 50% or more	44	46	38	42	50
Manipulative materials less than 1/4 of lessons	53	57	48	44	50
GRADE FIVE					
Single text	60	46	48	43	56
Whole class 50% or more	48	37	34	41	40
Manipulative materials less than 1/4 of lessons	81	84	78	82	82

h) Low Socio-economic Status Schools. During the past ten years, a great deal of attention has been paid and additional funds have been available to low socio-economic status (SES) schools. While 18 percent of the respondents indicated they taught in low SES schools, 66 percent of these second-grade teachers and 67 percent of the fifth-grade teachers had less than ten years' experience. Forty-eight percent of the second-grade teachers in these low SES schools were under 31 years old, and 38 percent of the fifth-grade teachers were. At the second-grade level, 59 percent of these teachers used a single text, 38 percent taught the whole class more than 50 percent of the time, and 46 percent used manipulative materials less than one-fourth of the lessons. At the fifth-grade level, 42 percent of the teachers used a single text, 31 percent taught the whole class more than 50 percent of the time, and 78 percent used manipulative materials less than one-fourth of the lessons. Even though the teachers in low SES schools, in general, were younger and even though they were faced with more complex problems associated with SES schools, there appeared to be little difference in approach or materials from other age groups or other SES schools.

i) Inservice. Finally, no real changes in methods or materials can take place unless teachers are given the opportunity for inservice training. The implementation of any new program bears a direct relationship to the understanding of the program by the teacher and the amount of assistance the teacher has. Patterns of such inservice support for curriculum innovation must remain the province of another study.

In this study, mathematics-related courses or workshops had been taken during the most recent school year by 32 percent of the respondents, but another 30 percent had not had such experience since 1970. Of these courses or workshops, 40 percent were sponsored by a district or a region, and another 27 percent by college extension.

In actual classroom assistance, only one-third had help from some kind of resource personnel, but two-thirds thought such help could be useful. Twenty-eight percent had never observed another teacher teach, and a total of 74 percent had observed other teachers at most four times. Yet 64 percent thought that such observations could help their teaching substantially.

Conclusions. The overwhelming conclusion to be drawn from these findings is that mathematics teachers and classrooms have changed far less in the past 15 years than had been supposed. The following is an over-simplification, but we think not a misleading description of the results.

1. The "median" teacher is a woman under 40 years of age who has been teaching ten years or less. She took two semesters of high school algebra, two of high school geometry, and two mathematics courses and one mathematics education course in college. She belongs to no mathematics teachers association and has observed someone else teaching a mathematics class at most two or three times (exclusive of master teacher, student teacher, and supervising teacher), but she believes that watching someone else teach children mathematics could improve her teaching. She is likely to find mathematics easy and interesting to teach.
2. The "median" classroom is self-contained. The mathematics period is about 44 minutes long, and about half of this time is written work. A single text is used in whole-class instruction. The text is followed fairly closely, but students are likely to read at most one or two pages out of five pages of textual materials other than problems. It seems likely that the text, at least as far as the students are concerned is primarily a source of problem lists. Teachers are essentially teaching the same way they were taught in school. Almost none of the concepts, methods, or big ideas of modern mathematics programs have appeared in this median classroom.

Here are some further conclusions:

1. If there are indeed declines in mathematics test scores, it is questionable that a large part of that decline can be attributed to "new mathematics" since

few of the reform movement's suggestions have been extensively implemented in the classroom.

2. Elementary teachers who specialize as mathematics teachers of self-contained classrooms, are a little better trained and have a somewhat more positive attitude toward mathematics teaching. They seem to make more use of the concepts and processes of the modern mathematics program.

3. If there has been increased funding and availability of additional help in the lower socio-economic status schools represented in this survey, these have apparently not resulted in great changes in teaching methods.

Other Studies on Use of Instructional Time. Over the years, studies have attempted to determine what a teacher does all day. Few studies, however, have been devoted directly to mathematics instruction. Recent studies by Conant [7], Olson [8] and Kelley [9] each give estimates of time devoted to mathematics instruction in grades K-6 and descriptions of how that time is spent. Exact figures vary from study to study. In the 1973 Conant study, done in Portland, Oregon with 47 teachers, approximately 100 minutes of the school day in grades 1-4 were considered related to instruction. Of these minutes, about 18 were in mathematics, and these were equally divided between individual and whole-class instruction. The 1970 Olson study was national in scope (112 districts) with a large sample (18,528 classrooms). It focused on instructional sessions. At the elementary level, 21 percent of them were devoted to mathematics. In the mathematics classes, seat work took up 39 percent of the time; questions and answers took up another 24 percent; and individual or small-group work, about 16 percent. Kelley's study in the Berkeley, California, school district in 1970 indicated approximately 37 minutes devoted to mathematics. He, like Conant, however, found that teachers spent a large amount of time with clerical or administrative duties. The result of the 1975 NCTM survey matched closely those of Kelley's earlier study.

At the secondary level, there appears to be a fairly consistent pattern in mathematics classes of 200-300 minutes per week. Olson's observations in secondary mathematics classes showed the dominant patterns were: question and answer, 33 percent of the time; seat work, 15 percent of the time; and individual work, six percent of the time.

Additional and continuing studies of classroom practice are needed to determine whether the traditional pattern of large percentages of instructional time devoted to teacher lecture and seat work on the same materials is changing as more alternatives are proposed.

Research Needs in Patterns of Classroom Instruction. The NCTM survey described above was designed and implemented in the face of a near vacuum of pertinent information. It provided NACOME with interesting, provocative data, but is obviously one small step. Much more information is needed. Many more questions need to be answered and many more teachers and other educational personnel surveyed. Primarily this study provides hints of what is happening in classrooms and, more important, it may be useful in suggesting fruitful directions for future investigation.

It should also be clear from this chapter that we need evaluation and comparative data on the variety of patterns in which instruction can be organized and studies of the interrelationship of variations in teaching and learning styles and their effect on achievement and other outcomes.

3.5 Summary

As students have individual learning styles, teachers also have individual teaching styles. To prescribe one style for all teachers is as mistaken as to affirm a single learning style best for all

students. Methods and organizational patterns and media are not panaceas. Teachers should be eclectic pragmatists, selecting those methods and materials which seem to work best at a particular time for a particular student or group of students working with a particular concept. There are times and situations for which large group instruction is still appropriate, just as there are situations, teachers and students best suited to small group or to independent work. Perhaps the optimum, judicious mix of all these modes is what the conscientious teacher seeks. It is unlikely that real educational progress results when groups lose all balance in passionate proclamations of their favorite teaching/learning patterns as THE WAY, exclusive of other patterns. The PR, ballyhoo, bandwagon approach to educational experimentation is counter-productive.

The recommendations we make are not specific to the teaching of any describable subset of children, whether gifted students in the ghetto or low achievers in the suburbs. It is obvious that any group defined has problems if not unique to itself at least exacerbated by its characteristics. But there are commonalities and generalizations that apply nearly universally as well. One recommendation for teaching children is just that -- that a wide variety of materials, instructional methods, and concepts should be available to <u>all</u> children. Each child has the right to be taught significant mathematics in a method consonant with his or her learning style by a well prepared, caring teacher. When we can assure each child this right, we will have made a giant step toward universal mathematical literacy.

CHAPTER 4. TEACHER EDUCATION

The dominant feature of the mathematics teacher education picture is the absence of hard data concerning programs and practices, requirements, and characteristics of the products. Much of what is written, discussed in conferences, and used to justify recommended programs is based on sketchy impressionistic data, random oases of innovative activity and research, and opinion. It is impossible to even attempt a description of the "typical" graduate of a teacher education pre-service program, much less that same individual after possible exposure to a wide variety of in-service training experiences.

Thus we begin with a caveat that the picture we present must be viewed with awareness of its limitations, for it is necessarily based on incomplete and even old information.

4.1 Pre-Service Education

Published recent data on the requirements in mathematics and mathematics teaching methodology of pre-service teacher education programs are non-existent. And the certification requirements of the states, while undeniably exerting considerable influence on the programs, are not reliable indices to what is actually contained in the institutional teacher education degree. At best they provide minimal, often vague, standards and a base from which inference is hazardous. At the secondary level, surveys have been conducted

recently and additional information should be available in the near future. But it is at the elementary level that variation in requirements, change, and pressure for change have been greatest and it is here that data are most sketchy and perhaps obsolete.

Prior to the decade of the 1960's the training of elementary teachers in mathematics was fairly uniform. Typically the teacher would take one course in the teaching of arithmetic and no mathematics content courses, unless a course was required as part of the institution's general education requirements for all students. (It should be remembered that a significant proportion of teachers now teaching in stable districts are products of such programs. The extent to which these teachers have been touched by in service courses, especially in content updating, is totally unknown.)

The high school teacher education requirement tended to be equivalent to the minimal mathematics major with other professional courses in methodology and educational theory. In fact, partly because mathematics teachers were in short supply, many persons teaching junior and senior high school mathematics did not have the equivalent of the training institutions' requirements for secondary school mathematics teachers. Little is known on a broad scale of the actual preparation of teachers now teaching high school mathematics.

In 1961, the Committee on Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America published guidelines for the training of teachers of mathematics. These were revised and updated in 1966. [1] There is evidence that these guidelines had considerable impact on the teacher education programs for elementary teachers. A plethora of new textbooks for college courses were published in the 1960's and nearly all of those related to mathematical content indicated in prefaces and introductions that they were based on CUPM guidelines.

In the early years of the 1960's CUPM held a series of forty-one conferences at state and regional levels on the training of teachers of elementary school mathematics. Teachers, mathematics educators, and state department certification officers participated. It is reasonable to assume that these conference exerted some influence on certification policies and teacher education institution curricula. While strict causation cannot be attributed to CUPM efforts, they were an important contribution to the general climate and pressure of the times to make teachers at the elementary level more knowledgeable in mathematics. Many changes in state certification requirements, teacher education requirements, and standards of the National Association of State Directors of Teacher Education and Certification can be documented during this period.

A survey conducted in 1966 by CUPM gives evidence of increased mathematics requirements by the teacher training institutions. There was a decrease from 1962 to 1966 from 23% to 8% in programs that required no mathematics credit hours and an increase from 27% to 38% in programs requiring 5-6 semester hours of mathematics. In fact, the mode remained at 3-4 semester hours (38.3%) but nearly as many institutions in 1966 were requiring 5-6 hours (37.6%). [2] At the least, these data can probably be interpreted to mean that the general pattern of change was for institutions to add one course to their mathematics content requirements. Such a change hardly seems monumental but when one recognizes that education in general and teacher education in particular is characterized by conservatism and inertia, this shift in attitute toward the mathematics component of an elementary teacher's preparation can be viewed as significant.

It is not at all clear that the CUPM recommendations had a similar impact on the preparation of junior high school and senior school teachers of mathematics, insofar as required courses are concerned. The role of advisors in planning of a secondary teacher candidate's program is a key factor, however, and it may be in this

way that the professional organization recommendations are more influential. The emphasis on the need for a modern geometry course, for computer science, for probability and statistics, for applications, that is found in recent recommendations, may be having more effect in course selection and in course design than statistical information could indicate. Detailed discussion of the CUPM 1961 and 1966 revised recommendations, as well as survey data, may be found in Chapter VII of the NSSE 69th Yearbook. [3]

At the time of the CUPM survey and conferences, considerable optimism was expressed that the pattern of change toward increased course requirements in mathematics would continue. It was felt that what the survey demonstrated was heartening but only a first step and that the momentum generated would lead to further, though gradual, change. Such optimistic predictions proved to be unfounded. It is doubtful that the situation today in pre-service mathematics teacher education is in any significant respect different than in 1966 and there are signs that many states and institutions may be backing off earlier support for a strong mathematics component. It appears that mathematics teacher education is in an embattled defensive position and may have difficulty in merely maintaining even its present portion of the total pre-service degree.

In the absence of recent survey information, one can only speculate from scattered and incomplete sources that the most typical pattern today in the pre-service program for elementary school teachers is one or two courses in mathematics covering number systems and perhaps some geometry and a methods of teaching mathematics course. The NCTM survey [4] tends to substantiate this although its results refer to all courses, not just pre-service. Of those responding, 18% reported having had one mathematics course, 32% had two, 31% had three or more. Sixteen percent had had no mathematics courses. For other courses in mathematics education, results showed 11% had none, 38% had one, 29% had two, 20% had three or more.

There is probably greater variation in the background of middle or junior high school teachers, some of whom moved from an elementary teacher's preparation with perhaps some additional mathematics course work, and some of whom were prepared as secondary teachers.

The senior high school teacher's content preparation is little changed from the 1960's except that more recent graduates are more likely to have worked with computers, are more likely to have taken courses in probability and statistics and perhaps combinatorics, and may have been exposed to some serious work in applications or modelling. Given that more of them will teach geometry than any content other than algebra or general math, it is likely that the weakest link in their content preparation is geometry. This is partially the result of the mathematics education profession's inability to achieve consensus as to the kind of geometry courses teachers should take and the failure of most mathematics departments to strengthen quantitatively and qualitatively the geometry segment of their offerings appropriate for secondary teachers.

In 1971, CUPM published an entirely new set of recommendations based on several conferences and its Teacher Training Panel's perceptions of the changing picture in mathematics curricula and updated assessment of the mathematical knowledge needed by teachers. [5] Like the 1961 and 1965 recommendations, the attempt was to deal only with the mathematics content component of the teacher's preparation. Rather detailed course guidelines were written. Major changes for elementary teachers were the following: (1) to stress the integration and interrelationship of concepts and the unifying ideas of mathematics, design the recommended courses in a spiral organization with gradual development from informal to formal, concrete to abstract; (2) expand and modify the informal geometry to incorporate considerable treatment of transformational geometry and coordinate geometry; (3) include some (nearly 20% of the detailed guides) probability and statistics; and (4) stress applications.

For high school teachers, additional emphasis was placed on applications, probability and statistics, and computer science. Course work in geometry was to include concepts in transformational, projective, and vector geometries as well as the more tradiational synthetic approaches. There is no evidence to suggest that these recommendations have had any impact on teacher education programs. In fact, they appear to have been largely ignored and little if any attempt has been made to apply professional pressure through conferences, accreditation or certification agencies. The difference in impact appears to be a matter of timing. There is no doubt that the 1961 and 1966 recommendations fell on very fertile ground. Attention to mathematical education in teacher training seems to be cyclical, ranging from positive emphasis to indifference. The climate of the 1970's has not been friendly to the concerns of mathematics teacher educators except as they fit the general trends and favored fashions in the field of professional education.

In 1973 the Commission on Education of Teachers of Mathematics (CETM) of the National Council of Teachers of Mathematics published Guidelines for the Preparation of Teachers of Mathematics. [6] The guidelines are stated in terms of specific competencies and cover mathematical content, contributions of humanistic and behavioral studies, teaching and learning theory with laboratory and clinical experiences, practicum and pre-certification teaching. Thus these guidelines are intended to cover the full range of pre-service preparation.

It is difficult to compare the CUPM guidelines with the Commission guidelines for content preparation because the formats are so different. There do not appear to be conflicts in the extent or emphasis on particular topics, however, and our judgment is that they are compatible. The entire set of four courses recommended by the 1971 CUPM guidelines for elementary teachers probably entail considerably more mathematical content than can be inferred from the

NCTM Commission competencies, and the great stress placed by CUPM on unifying and integrating ideas is not apparent in the Commission guidelines. There is greater emphasis in the Commission guidelines on the practical use of instruments (e.g. use of the calculator in problem-solving) than is to be found in the CUPM document.

There have been very few well-publicized programs experimenting with changes in the pre service mathematical education of teachers in recent years. A notable exception is the Indiana University Mathematics Methods Program which integrates content, methods, laboratory clinical and field experience within course modules, rather than separating them in the traditional manner. Teacher educators have expressed considerable favor for the concept of integrating content and methods. But there has been limited implementation in formal program structures. The difficulties of cutting across administrative and departmental lines in colleges and universities have blunted many attempts at organization change. On the other hand, it appears that a great deal of methodology is inserted into content courses, and vice versa, when the instructors are knowledgeable, competent, and interested in both the mathematics and the pedagogical and curricular concerns.

Over recent years there has been a significant change in methods courses through the introduction of mathematics laboratory experiences. (The laboratory method is sometimes utilized in the content courses as well.) As physical materials for use in mathematics teaching have become increasingly available, exploration and investigation of materials and activities in laboratory settings have become commonplace in the elementary teacher education program. In some instances, the laboratory carries the major burden of instruction for pre-service elementary teacher candidates.

There is little evidence of recent widespread change or innovation in the methods component of the secondary teacher education

program. Perhaps an exception is the growing use of video taping. In 1972 a survey of mathematics educators was carried out to determine recommendations for secondary teacher education. [7] A large number of changes in practice were suggested. A near consensus was found on the need for earlier and more extensive observation and field experience prior to student teaching. We believe this to be a need at elementary level also. In the NCTM survey, 28% of the teachers responding had never observed another teacher teaching mathematics except as part of their student teaching experience.

The NCTM Commission on Education of Teachers of Mathematics has indicated several serious concerns about current pre-service teacher education. One is the severe lack of research on and evaluation of teacher education programs and especially the need for shared information and coordination of research and evaluation efforts involving several teacher training institutions. Another is the recommendation that earlier field experience be incorporated into programs in such a way as to accomplish the overall goals and objectives of the program. The Committee emphasizes the need to relate research and teacher education efforts in mathematics teaching to the present emphasis on preparing special education teachers for the handicapped and for children with learning disabilities. Specifically, joint efforts between the mathematical education community and the specialists in educating children with handicaps or learning disabilities should develop guidelines for mathematics in special education.

4.2 Certification and Accreditation

Certification of teachers is the responsibility of the 50 states and the District of Columbia, and there is wide variation in the degree of specificity of requirements. Many states put requirements in such a way as to leave latitude for interpretation by the teacher education institutions and some base certification on completion of a NCATE (National Council for Accreditation of Teacher Education) accredited program. There is no common or typical pattern in the mathematics requirements of state certification.

The organization to which state certification agencies belong is the National Association of State Directors of Teacher Education and Certification. It publishes general standards which individual state agencies support. The 1971 NASDTEC standards for college programs preparing secondary mathematics teachers are:

> STANDARD I. The program shall consider the sequential nature of mathematics and shall provide the prospective teacher an understanding of some of the aspects of mathematics which his pupils will meet in subsequent courses.
>
> STANDARD II. The program of preparation shall include mathematical studies and experiences which are relevant to the school curriculum e.g., algebra, geometry, trigonometry, analytic geometry, calculus, probability and statistics.
>
> STANDARD III. The program shall assure knowledge by the prospective teacher of curriculum improvement studies in mathematics currently being made by various national groups.
>
> STANDARD IV. The program shall provide knowledge of ways to apply the principles of mathematics to other disciplines, e.g., logic, science, psychology, economics.
>
> STANDARD V. The program shall develop for the prospective teacher an understanding of the historical relationships of mathematics to the culture in which it existed or exists.
>
> STANDARD VI. The program shall provide laboratory experiences in working with pupils of both high and low academic abilities and shall develop the ability to teach computational as well as abstract mathematics.
>
> STANDARD VII. The program shall include a substantial experience in the field of computing as it relates to mathematics and the teaching of mathematics.
>
> STANDARD VIII. The program shall provide substantial experience with mathematical model building.

In the field of accreditation, the regional associations accredit institutions. Academic areas have accrediting agencies specific to those areas. In teacher education the accreditation agency is the National Council for Accreditation of Teacher Education (NCATE). The standards of NCATE do not speak specifically to the question of mathematics or mathematics methods requirements but one standard requires that the institution demonstrate it is familiar with and has considered the guidelines of professional organizations.

The guidelines of NCTM's Commission on Education of Teachers of Mathematics and of the MAA Committee on the Undergraduate Program in Mathematics are the pertinent guidelines to be used in evaluating mathematical education components of teacher education programs. Reports from institutions having had NCATE team visits suggest, however, that there is considerable variation in the extent to which this standard is investigated. For many visits it seems to be sufficient for the institution to list guideline documents on hand with little attempt by NCATE to check on whether the institution's decision makers are familiar with the contents. If no NCATE visiting team member represents mathematics, for example, it may be difficult for team members from other disciplines to interpret the specifics of the guidelines and the institution's work is taken at face value. An attempt by NCATE (the PEP plan) to ask for subject matter experts' separate advice on institutional reports has recently been abandoned.

The Mathematical Association of America has produced a set of guidelines for use by regional accreditation associations in evaluating mathematics departments and these guidelines speak to the question of the mathematics course offerings for teacher education candidates. These guidelines were revised in 1975 by a committee with representatives from both MAA and NCTM. [8]

These guidelines, it should be noted, pertain to the minimal course offerings expected of mathematics departments and to the quality of instructors for these courses and are not directly concerned with requirements of the teacher education degree. They do, however, present professional opinion as to the content of courses provided for teacher education programs. They state that for an institution that prepares elementary teachers, the mathematics department should provide at least 3 courses designed appropriately for elementary teachers. The statement is significant in that the professional organization in mathematics (MAA) is saying that courses should be developed around the specific needs of elementary teachers,

that not just any of the department's regular offerings are appropriate. Content should include at least what might be called the "theory of arithmetic," informal geometry, and probability and statistics. The guidelines also state that instructors in such courses should be "prepared and competent in mathematics, but with experience and interest in elementary school curricula."

Courses provided for the secondary teacher training program should include linear algebra, abstract algebra, probability and statistics and geometry, electives from a prescribed list of courses, and a course dealing with modeling, applied mathematics or programming.

What might turn out to be the most important change in teacher education certification and thus teacher education programs in the near future, Competency-Based Teacher Education or Performance-Based Teacher Education, will be discussed in a later section on general trends in teacher education.

4.3 In-Service Education

In recent years the attention of mathematics teacher educators has turned from pre-service to in-service education. This is almost exclusively true for the two committees within NCTM and MAA which are those professional organizations' arms for teacher education concerns. The activities of CUPM related to teacher education are now in the in-service realm. The Commission on the Education of Teachers of Mathematics conducted a survey in 1974 of classroom teachers' perceived in-service needs. [9] Both committees are at present planning in-service projects.

The Commission recognizes that the sample for its 1974 survey is not entirely representative. NCTM membership roles were used to select the sample and obviously such individuals are the more professionally-oriented among the teacher population. Some of the

samples' characteristics bear this out. The average respondent had eleven years teaching experience, tenure, course work or degrees beyond the bachelors degree and taught in an urban or suburban junior or senior high school. Nonetheless, some useful information was gained. Topics ranking high on the list of those in which teachers feel a need for in-service work are: motivation, metrication, laboratory learning, slow learners, learning styles of students, mathematical games, development of instructional materials, problem solving, classroom use of computers, applications, gifted learners. Those topics were checked by over 100 of the 266 respondents. Lowest on the list are content topics. It is of interest that of the content topics the one that ranks highest in perceived need for work is geometry, checked by 76 respondents.

In their attitudes toward in-service education, 88% felt a need for in-service work but 61% felt in-service education has not fit classroom needs and 37% reported the in-service work they had had to have been "a waste of time and energy." Three-fourths believed that teachers should be released from classroom duties for in-service education and 76% thought they should be paid expenses for such work; 72% thought in service education should be required for mathematics teachers and 90% believed school districts should make it available. A small majority preferred that in-service education be separate and distinct from university degree programs.

The Commission rightfully calls attention to the trend toward teacher demands to participate in the design of teacher education programs, teacher certification and program accreditation and the trend toward school district reliance on their own resources (perhaps with outside consultative help) in the design and implementation of in-service education. The almost absolute control by teacher education institutions of the training of teachers is being weakened, and realignments in the control and influence of various groups within education on teacher education and entry into the profession are

certain to occur in the near future. The implications of such political changes for both pre-service and in-service teacher education are profound.

The governmental funding agencies, NSF and NIE, currently are putting the emphasis within their teacher education efforts on in-service programs. Very little funded experimentation is to be found in pre-service programs.

At the state level there have been some examples of successful in-service projects, notably the Specialized Teacher Project in California. The project was established by the California State Legislature and provided in-service training for elementary school teachers. Teachers attended two-week summer workshops organized along the lines of the Madison Project and designed "to acquaint teachers with the techniques of creating in their classrooms a climate in which pupils might more effectively learn mathematics." [10] Orientation was toward manipulative materials activities and inquiry-discovery methods. Content emphasis was from the Mathematics Framework for California Public Schools (The Second Strands Report): numbers and operations, geometry, measurement, applications of mathematics, statistics and probability, sets, functions and graphs, logical thinking and problem solving.

Evaluation for 1971-72 supported earlier (1968-69, 1969-70, 1970-71) findings, that the pupils whose teachers attended the workshops perform significantly better on measures of comprehension and computation than pupils whose teachers did not receive the training. Statistically significant growth on measures of mathematical achievement was reported by pupils of the teachers who attended the workshop. In looking at longitudinal effects, evaluation found that attendance of a second summer workshop produced a substantial improvement in pupil performance, particularly in measures of mathematical comprehension, and that workshop had a lasting beneficial

effect that carried over to a third year even though their teachers did not take a workshop the third year.

In 1973 a summer workshop program patterned after the Specialized Teacher Project in California was begun in the State of Washington. This program has the special feature that the workshop participants are required to give in-service training to other teachers in their school buildings during the school year following the workshop. This proliferation scheme has the potential for greatly extending the benefit of the workshop experience. The project is continuing and evaluative information is not yet available.

There is considerable activity in in-service work at local levels and systematic information needs to be collected on the extent of these efforts. Information is from scattered sources and the overall picture is not clear.

One increasingly popular format for in-service work is the Teacher Center. Centers may be operated solely within a given school district or in some cases are cooperative efforts that include several districts. One salient characteristic of Teacher Center activities in teacher training is the central role played by the teacher in assessing needs, designing and controlling programs. To some extent Teacher Centers are the teaching profession's own attempt to update and maintain its preparation. However, it is nonetheless true that in most instances, Centers are organized and funding secured by college teacher educators who act as consultants or by district administrators. But subsequent administration and governance is often in the hands of the teachers, at least in the capacity of governing advisory bodies. For a comprehensive look at the present status of the Teaching Center movement see a report by Schmieder and Yarger for the American Association of Colleges of Teacher Education. [11]

There is pressing need for a continuing program of in-service education, both in large scale systematic efforts and in small individual ways such as teachers sharing of ideas or observing other teachers. But more is required than the initiatives of teachers, funding agencies and professional organizations. School district administrators must have a serious commitment to continuing teacher education responsibilities and provide the means in time and funding to enable teachers to take advantage of in-service education opportunities whenever available. These opportunities go beyond the organization by the district of formal course work or workshops.

On the whole, curriculum development projects have paid insufficient attention to the education of teachers essential to the lasting success of their products. Their efforts have often been confined to minimal training in how to use the project materials and methods. It is doubtful that two-week or even two-month workshops will have any significant impact on the fundamental proparation needed by teachers to assure improvement in mathematical instruction.

4.4 General Trends in Teacher Education

It is doubtful that the present situation in mathematics teacher education can be understood without a fairly clear perception of the major trends in teacher education in general.

The clearly discernible shift in attention from pre-service to in-service teacher education on the part of the mathematical education community and the funding agencies may have its roots in two fundamental trends. One is a major political movement in decision making and governance in teaching as a profession; the other is the present practical economic reality of teacher supply and demand.

As was mentioned earlier in this section, as teachers as a profession increasingly find strength in organization and grow more assertive, there will be increasing demand for more meaningful

participation in the governance of education, of teacher education and of entry into the profession. Initially the natural entering point into decision-making is at the level of in-service education, which has been traditionally more localized and subject to the control of school districts. The next logical step is probably in the process of certifying individuals for entry into teaching. But the latter process has a significant influence on pre-service program requirements.

The broad outlines of the job market situation are familiar to everyone. We have come from a situation of shortage to one of over-supply of teachers. But the details and what they entail for the future change rapidly. Data are available concerning teacher supply and demand and teacher turnover but become obsolete rapidly. These data, furthermore, do not give a detailed picture of the situation in particular fields of teaching, such as mathematics.

In 1973 and 1974 the Rand Corporation did a study for the Department of Health, Education, and Welfare, Analysis of the Educational Personnel System. Models were developed to make projections of teacher supply and demand. The Rand study, like other projections, forecasts a continuing over-supply of teachers until at least 1980 but the analysis cast some doubt on the magnitude of other projections. It suggests that the dynamics that produced the surplus are changing and that it appears we might reenter a period of shortage in a decade or so. [12]

In particular, the Rand analysis suggests that earlier studies overprojected future supplies of new teachers because now the rate of decline in the production rates is likely to be significantly larger than the rate of growth in numbers of new graduates. Thus Rand expects a sharp decline in the annual supply of new teachers throughout the 1970's.

As the Rand study points out, there is "considerable inertia in the supply of teachers." Current levels of production depend upon choices made perhaps four years ago. It reminds us that the inertia works in the opposite direction as well. In summary, the provocative prediction made by the Rand analysis is that "it appears that if and when the surplus ends, the inertia in the system will lead to the almost immediate onset of a substantial and lengthy teacher shortage." [13]

While economic realities undoubtedly justify an immediate concentration upon in-service education in that it may be more economically efficient to retrain present teachers than to train new ones, it will probably be short-sighted and extremely costly in the long run if the inadequacies of present pre-service education are ignored. It is apparent that the mathematical education community, through its professional organizations considers present pre-service programs far from ideal. Yet a sufficient base of support for experimentation with changes in pre-service teacher education cannot be seen as forthcoming.

It is impossible within the boundaries of this report to discuss adequately the many trends in teacher education which bear upon mathematics education and to which mathematics teacher education must constantly react and accommodate itself. The reader is referred to a summary of these trends prepared by J. Myron Atkin and James D. Raths of the University of Illinois, Urbana, Illinois [14]

The trend discussed at length by Atkin and Raths and likely to have the most profound consequences for teacher education in the immediate future is competency-based teacher education (CBTE) or performance-based teacher education (PBTE). In a 1972 survey by the American Association of Colleges of Teacher Education (AACTE), 1250 institutions were asked if they were operating, investigating and/or planning competency-based programs. Of 783 respondents, (63%

return), 17% said they currently operate such programs, 29% said they were not and 54% said they were in some stage of investigating.

But the real push for incorporating CBTE in the training institutions will come from the state department level, where interest and activity in competency-based certification runs high, or from state legislative action. In November, 1974, the Multi-State Consortium on Performance-Based Teacher Education published in its newsletter a survey of activities at state level on competency-based teacher education. [15] Most states responded in terms of the use of the concepts in certification.

To summarize the responses reported from 50 states and the District of Columbia: 5 have some form of competency-based certification in use either for initial or recertification or as an alternative; 23 report some definite official action already taken to move toward CBTE or competency certification; 23 report they are in some stage of investigating or studying the concept. Every state indicated a positive interest.

CBTE or PBTE is highly controversial with passionate proponents on one side and hostile opponents on the other. It is the manifestation in teacher education of the accountability pressures on education. Basically the idea is to identify the operationally-defined skills that relate to student achievement and to build the education of teachers and the gatekeeping rules for entering the profession on these "competencies". It is a part of the inchoate feeling on the part of the public that somehow education is not accomplishing its goals (whether realistic or not) and that this failure can be rectified by efficient management. Atkin describes the response reflected in CBTE and other managerial systems as the adoption of the engineering model. He claims that the engineering metaphor undergirds the thinking of all those who see the educational system as a mechanism to be redesigned to meet certain agreed upon and prespecified goals. [16]

Critics argue that the atomization necessitated by listing specific competencies ignores essential interactions, eliminates higher-order concepts and processes, chooses only low-level skills easy to measure, considers only short-term objectives, reduces teaching to a routinized, mechanical task and generally trivialized educational goals and processes.

Without entering the argument, one can say two things objectively; first, there is totally lacking a research base to support the concept, and second, its serious implementation would almost entirely change the character of teacher education. Reviews of the literature on PBTE conclude that there is no empirical support at present for the concept. Heath and Nielson state, "In our opinion, an analysis of the research on the relation between specific teacher skills and student achievement fails to reveal an empirical basis for performance-based teacher education." They conclude that the conception, design and methodology of the studies "preclude their use as a basis." More serious is their criticism that the PBTE model does not recognize two important types of variables. "It ignores what is to be taught: and "the model ignores who is to be taught." We agree with their opinion that "It seems unlikely that one set of teacher behaviors is most effective for teaching everything to everybody." [17]

The NCTM Commission on the Education of Teachers of Mathematics has reported it will not take a stand until more evaluative evidence can be provided. This is no doubt an objective and scientifically sound position but has resulted in silence on the part of NCTM as an influence outside its own membership on a highly sensitive and political matter which has serious implications for mathematics education.

As a more politically sensitive act, the National Council of Teachers of English passed a resolution urging officials in education

not to move exclusively into CBTE or competency-based certification until more research and objective evaluation of its outcomes are available.

The members of NACOME also take the position that neither teacher education nor certification procedures should be based solely on competency or performance-based criteria without a sound empirical rationale. Furthermore, we believe that the crusade-like zeal and bandwagon mentality with which these concepts are sometimes promoted and accepted is a real and present danger to mathematics education. We urge responsible authorities in both public and private sectors to insist upon sound and objective justification before embarking upon a course with such profound implications.

There are undoubtedly positive results possible from a thorough dispassionate, objective study of what competencies, abilities, skills, and knowledge relate directly to effective successful teaching. Clarification and agreement on broad goals for mathematical education are, however, a necessary condition and prerequisite to determination of such competencies. At present, we seem to be a long way from these ideas. Reasoned and responsible investigation and argument from both sides of the issue are sorely needed.

4.5 Teacher Effectiveness

The issue of competency-based criteria is embedded in a larger problem, that of inability to define clearly and thus identify "good" teaching. Are there identifiable teacher characteristics that bear a relation to teacher effectiveness? What shall be the criteria for judging effectiveness in teaching?

There is a vast body of research that attempts to come to grips with these and similar questions. Reviewers of this long history of research conclude that an educationally-significant relationship between teacher characteristics and student achievement simply has

not been demonstrated. The problem is terribly complex and is perhaps beyond our present level of research methodology. But there is no doubt that it is of crucial significance to every segment of the educational system and in light of its importance it still seems to merit concerted and large-scale efforts toward some stage of empirical sophistication. The alternative is to continue to judge teaching on a merely subjective, vague or even ineffable basis. We urge continued and fresh, creative efforts to define effective teaching and the successful teacher and to develop methodology useful in identification and recognition of good teaching models.

CHAPTER 5. EVALUATION

The most important indicators of quality in school mathematics curricula and instruction are the measures of student achievement and attitudes. Most recent critics of school mathematics have supported their arguments with selected data indicating a sharp decline in student performance on nationally standardized achievement tests. The declining mathematics test scores are only a part of general school achievement trends that also include weakened performance in reading, writing, and science. However, to examine the validity and implications of the "test score" issues, NACOME has collected achievement data from four major sources: developers of major nationally normed achievement tests, mathematics assessment efforts in several states, the National Assessment of Educational Progress, and research studies such as the National Longitudinal Study of Mathematical Abilities.

The questions to be answered by investigation of achievement and attitude data are deceptively straightforward:

> Are students acquiring the mathematical skills and understanding needed for meaningful participation in our contemporary technological society?
>
> Is school mathematics successful in preparing interested students for careers in business, industry, government, engineering, and the sciences like physics, biology, statistics, or computers?
>
> Are students developing positive attitudes toward the methods of mathematics and appreciation of the role that mathematics has played in development of contemporary culture?

Unfortunately, there has been no comprehensive long-term effort to measure the effectiveness of American mathematics education in achieving these goals. As NACOME attempted to piece together a profile of attitudes and achievement among students in grades K-12, the data fitted no clear or simple pattern and suggested much deeper questions about the nature of mathematical ability and the relation of assessment techniques to goals for school instruction. Recent trends in achievement vary according to grade level, geographical region, and instructional emphasis. Furthermore, results in a single grade, region, and curriculum vary over time -- undoubtedly reflecting changing interests of students and priorities for schools.

The information and recommendations of this chapter are presented in two main sections. The first section is a collage of available achievement data; the second section is a discussion of the nature and assessment of mathematical abilities, examining the adequacy of current and proposed measurement schemes and the impact that assessment has on curriculum and instruction.

5.1 Achievement Data

Information on the current mathematical achievements of students K-12 was available to the Committee from several sources -- each focusing on somewhat different goals and employing different measurement procedures.

State Assessment Reports. Though many state departments of education now regularly test the mathematics achievement of students K-12, only two states (New York and California) have conducted such programs long enough to give reasonable indications of meaningful trends.

In 1966 the New York State Department of Education initiated a program of annual fall achievement testing required for all pupils in public and non-public schools. Originally used for allocation of

Federal Title I funds, the program established levels of performance in reading and mathematics at grades 3, 6, and 9 that identified the lowest 23% of 1966 students. Subsequent test results have been reported as "percent of students falling below the 1966 reference point". By 1973 performance of New York students had changed noticeably. [1] Only 18% of the state's third graders fell below the previous 23% cutoff point. However, at grade 6, 32% of the students and at grade 9, 34% of the students performed below the 1966 reference point. Thus in some sense mathematical performance improved in the third grade and declined in the sixth and ninth grades from 1966 to 1973.

As might be expected, the greatest changes were in New York City and other large cities, with the third grade increase and sixth and ninth grade declines being greatest in the cities. Furthermore, most of the change occurred between 1966 and 1970, a period of political and racial stress in large city schools of the state. The trends in reading achievement were generally the same as those in mathematics.

Results of the California-mandated testing show a decline in mathematics achievement during the years 1969-1973. [2] In grade 6, the median score on the Comprehensive Tests of Basic Skills (CTBS) mathematics test dropped from 47 in 1969 to 38 in 1971 and then remained constant for the next two years. In grade 12, the median score on the Iowa Tests of Educational Development (ITED) mathematics test showed a modest decline from 49 to 46 over the five-year period. During this period the California reading scores on these tests declined also.

In 1971 and 1972, California implemented the SCIMA testing program which provided statewide mathematics achievement data for a variety of content areas at several cognitive levels. At grades 3 and 6, there were no significant differences in any category between 1971 and 1972 scores. However, at grade 8 there was a significant

decline in the content areas of whole numbers, rational numbers, and mathematical sentences. [3]

For many teachers and parents, declining mathematics performance simply means declining skill at arithmetic computation. Alarmed by indications of falling computational skill scores, the New Hampshire State Department of Education conducted several longitudinal studies of mathematics achievement during the 1960s, a period of implementation for newer mathematics programs in that state. [4] The results are difficult to interpret, since the groups of students being compared were selected solely on the basis of the textbook used for instructional purposes and different tests were used in different years. Groups of students using modern, transitional, and traditional tests were compared in 1965 and 1967. In 1965, the modern group scored significantly higher on the Otis Gamma Mental Abilities Test than either of the other two groups, but lower on the arithmetic computation test. In 1967, the modern group scored higher on the computation, concepts, and application subtests as well as on the Otis tests. However, all three groups showed declining ability to perform arithmetic computation.

Two more New Hampshire studies compared the performance of 1965 eighth graders two years later when they were in tenth grade studying algebra and geometry. At this stage the groups did not differ markedly on ability measures, the transitional groups was superior to the other two on a test on numerical competence, and the modern and transitional groups were dramatically superior to the traditional group on a test of algebra and geometry achievement.

The New Hampshire studies have been interpreted both as indictment and support for the innovations of "new math". But what comes much more clearly from reflection on the data is a realization of the complexity of curricular innovation and its evaluation. The sharp drop in performance on traditional arithmetic skill goals,

independent of curriculum embodied in textbooks, suggests a difficult transitional period as elementary school teachers tried to learn new mathematical ideas and begin implementing them in their teaching. Furthermore, evaluating mathematics programs with tests normed according to different goals and priorities clearly gives misleading indications of effectiveness. The orientation of teachers, students, and parents to language, concepts, and methods of new curricula is a slow process.

Though New York and California are the only states with records of student achievement over many years, other states have accumulated achievement data in mathematics for several recent years -- after curricular innovations have had some time to settle in. Recent results from Rhode Island, Delaware, Mississippi, and Virginia [5] suggest steadily improving mathematics performance in those states. In several other states that have completed only one round of assessment or have adopted a criterion referenced style of testing, the performance record in mathematics closely parallels that in reading suggesting that subject matter achievement is bound up in a complex web of school characteristics.

Standardized Test Trends. Until recently, the most common national indicators of school mathematics performance have been the variety of college entrance examination tests. Increased state and local testing at elementary and junior high school grade levels has now made standardized test batteries such as the Metropolitan Achivement Test, Iowa Tests of Basic Skills, Comprehensive Tests of Basic Skills, or California Achievement Tests equally well known as common denominators of school achievement. NACOME examined these nationally standardized mathematics tests to get insight into school curricular trends, but we also sought from test developers data and informal insights into national achievement trends.

The pattern of results for the Scholastic Apitude Tests, used for college admission decision making, is unmistakable and widely

known. [6] From 1962 to 1975, the mean score on the quantitative section of the SAT has declined each year. The total drop has been from a high of 502 to the present 472. At the same time, there was a drop in the mean verbal score from 478 to 434. Perhaps more significantly, the percentage of scores above 600 in mathematics declined from 20.2 to 16.4 while the percent of verbal scores in that range declined from 14.6 to 8.9. These results clearly show that for the upper secondary level, the decline in mathematics abilities was not as marked as the decline in verbal abilities. What is not so clear is whether or not the mathematics score decline would have been more severe if the reform movement had not occurred. However, it is interesting to note that the scores on the College Board Level I and Level II achievement examinations in mathematics did not exhibit this decline.

Developers of the standardized achievement batteries used in elementary and junior high schools collect national samples of student performance only when establishing norms for new versions of the tests. The norms are usually established by performance of roughly 2,000 students at each grade level, chosen by carefully stratified sampling procedures. When a new test is "normed", it is common practice to administer the test version being replaced to an equal sample, in order to assess comparability of performances. The most widely used achievement batteries are the Iowa (ITBS) and the Comprehensive (CTBS) Tests of Basic Skills. Reports from developers of these tests indicate recent decline in performance of norming groups.

The Iowa Test has concept and problem solving sections. Between 1963 and 1970 ITBS data indicate general improvement in the lower grades, but consistent and sizeable losses in the upper grades on both concepts and problem solving. Fairly consistent losses also occurred in reading and some language skill areas during the same time period. [7] The CTBS consists of computation, concepts, and

problem solving sections. Between 1968 and 1973 performance on the mathematics computation section of the test dropped sharply; the drop was accompanied by a smaller decline in verbal scores. [8]

A number of local studies have also verified the decline in achievement of basic skills during the late 1960's and early 1970's For example, a study of the Stanford Achievement Test scores of fourth graders in a modern mathematics program in a New Jersey school system for the years 1968-1972 showed a decline in mathematics subtest scores as well as comparable decline in the subtest scores for word meaning, paragraph meaning, spelling, work study skills, language, social studies, and science. [9] This study also showed that the timed nature of the arithmetic computation subtest significantly affects the performance of these students on the subtest and suggests that students in a modern mathematics program may be able to compute as effectively but more slowly than students in a traditional program.

In contrast, Milton W. Bechman administered a basic mathematical competency test to a group of 1,296 students in 42 Nebraska high schools in the fall of 1950. [10] In 1965, the same test was given to a comparable group of 1,384 students from 40 of the same high schools. The mean score of the 1965 students was significantly higher than the mean score of the 1950 students.

The National Longitudinal Study of Mathematical Abilities. As soon as the experimental curriculum materials of SMSG, UICSM, UMMaP, and the Greater Cleveland Mathematics Projects entered schools for pilot testing, teachers and educational researchers began a series of small scale comparative studies: new math versus traditional math. Using year-end mathematics achievement as the criterion of success, these studies usually showed that the two groups performed at about the same level on standardized tests. Sometimes students of conventional textbooks were slightly better at doing arithmetic

computation. If a test was given that used modern terminology and that dealt primarily with concepts unique to modern programs, students using the modern textbooks showed a strong superiority, as one would expect.

Although these studies may have quieted the fear of alarmists who predicted drastically inferior achievement by students of the new materials, there was a clear need for more extensive investigation with the following characteristics:

1. To separate the effects of using a particular textbook from the effects of having a particular teacher or being in a particular school, one needed a study that involved many teachers in many schools.
2. To determine the differential effects of different curricula, one needed a testing program that measured achievement in many potentially independent content and ability dimensions of mathematics.
3. To test the contention of new math developers that many of their fundamental learning goals would develop only after a student had been in the newer programs for several years, one needed longitudinal research.

In 1961 SMSG undertook such an investigation, the National Longitudinal Study of Mathematical Abilities (NLSMA). [11]

In the fall of 1962 NLSMA identified and began testing three groups of students: The X-population, consisting of 38,000 fourth graders in 1962, was tested every year for five years; The Y-population, consisting of 48,000 seventh graders in 1962, was tested every year for five years; The Z-population, consisting of 24,000 tenth graders in 1962, was tested every year for three years. All subjects were from schools that volunteered for the study and SMSG had no control over the textbooks or instructional methods used in the classes.

At the outset of the study, mathematicians of the SMSG Panel on Tests hypothesized that there are many components of mathematical achievement and ability rather than a single unitary trait. Thus the

NLSMA test batteries administered each fall and spring during the study consisted of numerous short scales -- each designed to assess an identified component of mathematical achievement. These scales included measures of student achievement on specific mathematical topics, as well as development of various psychological traits such as mental ability, attitudes toward mathematics and school, spatial visualization and deductive reasoning and self-concept. Questionnaires sent to schools and teachers gathered data about the instructional and socio-economic settings for learning.

The main purpose of NLSMA was to investigate the relationship of textbook usage and the broad collection of mathematical ability, achievement, attitude, and psychological variables measured in the study. The NLSMA approach to comparing textbooks placed less emphasis on the question "Which is best?" and more emphasis on the question "What are the patterns of achievement associated with the use of various textbooks?" NLSMA investigators were not in a position to control the textbook usage in participating schools, and the diversity of textbooks used during the five year period is testimony to the changes taking place in school mathematics and the independence of local school districts. In all, over 800 different textbook series were in use at one time or another during the five year study. To facilitate comparisons, only textbooks used in several schools were included in the analysis. Grades 4, 5, and 6 were grouped for the analysis; grades 7 and 8 were grouped; and each of the grades 9-12 was analyzed separately.

The main conclusions of the study at grades 4, 5, and 6 are the following: First, after achievement measures have been adjusted for a reasonably comprehensive set of initial conditions, many significant differences between textbooks groups remain. Second, between the behavioral levels of computation and comprehension there were radically different patterns of achievement among the textbook groups. For instance, for those measures classified as computation, the SMSG

group was below the grand mean for nine of the 11 scales. On the other hand, at the behavioral level of comprehension, the SMSG group was above the grand mean on all of the 19 scales and indeed had the highest standardized adjusted mean for 14 scales. This tends to reinforce the contention of SMSG test planners that mathematical achievement has many partially independent components. Furthermore, as E. G. Begle has said, SMSG students could "add well enough to win the problem solving contest". [12]

From a somewhat different point of view, the study showed that while there is a clear trend for modern textbooks to be associated with poorer performance on computation scales, the modern textbooks themselves are associated with widely varied patterns of performance on the other scales. This observation reinforces our earlier caution against viewing "new math" as a uniform, easily identified program of concepts and skills.

At the seventh and eighth grade level, the NLSMA analysis suggested the following general conclusions. First, students who used a conventional mathematics textbook series in the seventh and eighth grades did relatively well on test items dealing with computation and relatively poorly on test items that require the more complex abilities of comprehension, application, and analysis. Though the SMSG group achievement fitted the opposite pattern, the different modern textbooks exhibited quite dissimilar profiles of performance on many of the mathematical scales. Second, the results of both X- and Y-population analyses at grades 7 and 8 support one of the guiding hypotheses of NLSMA: that mathematics achievement is a multidimensional phenomenon. Evidence for this assertion comes again from the split between computation scales and higher-level scales. Unfortunately, the concentration of scales in the number systems category did not permit an adequate test of the model at the seventh and eighth grades.

While "new math" programs in elementary and junior high schools were accused of sacrificing computation ability in order to boost comprehension, similar criticisms were directed at high school algebra instruction. Results of testing the Y-population in grade 9 give some support for this contention. In general, the modern textbook groups were relatively strong in ability to deal with number properties, graphs, and algebraic inequalities and relatively weak in ability to perform routine algebraic manipulations and to formulate number sentences. There was substantial variation in performance levels within the set of modern text groups, although overall there is evidence that the modern textbooks do promote comprehension. However, the question of whether the modern texts promote higher-level thinking may have to wait for more sophisticated measures than were used in NLSMA.

In general the analyses for grades 10, 11, and 12 showed fewer clear trends of achievement associated with textbook classification. Among possible explanations for this situation, most plausible is the fact that senior high school teachers are generally far better trained in mathematics than their elementary or junior high school counterparts. Thus they tend to rely less on textbooks for course outline and emphasis -- excising and enriching textbook presentations where they see fit.

The preceding discussion has focused on relations between textbook usage and achievement. Though NLSMA also obtained extensive data on attitudes and many psychological attributes of students (such as anxiety or flexibility), the study revealed few relationships between these variables and basic mathematical achievement. These findings do not necessarily mean that no such relationships exist. More likely, the measuring instruments for affective variables may not have been appropriate or sufficiently sensitive.

As NLSMA date collection was being completed, SMSG undertook a similar, more limited investigation of textbook usage and achievement

in grades 1-3. The Elementary Mathematics Study (ELMA) did uncover several interactions between textbook style, socio-economic status of students, and patterns of achievement. However, the results were much less definitive than those of NLSMA.

National Assessment of Educational Progress. The most recent national indicators of the mathematical abilities being acquired by students in elementary and secondary school are the 1975 reports of the National Assessment of Educational Progress. Based on testing done during 1972-73, the NAEP reports describe achievement of 9, 13, and 17 year olds in 15 content strands at six levels of behavioral complexity. For a mathematical knowledge or skill to have been included in the assessment, it had to have been considered by scholars, laymen, and educators as something that should be taught in American schools.

The exercises developed for use by National Assessment differ from the more common educational test items in that these exercises are not designed to discriminate between individuals or groups. The concept of total score is inappropriate for interpretation of NAEP results. In fact, the main technical report on mathematics is simply a reproduction of released items with description of student responses to each item. For instance, [13]

Problem: Add

$ 3.09
10.00
9.14
5.10

Responses:	9	13	17	Adult
Correct answer	40%	84%	92%	86%
Added correctly, but made decimal error	22%	8%	2%	6%

While it is helpful to get such detailed information on mathematical abilities -- rather than the single scores reported from use of traditional standardized tests -- interpretation of the results must be approached with a good deal of care. Many mathematics

educators have been pleased by the performance of 13 and 17 year olds on the above sample problem. Yet others express shock that 14% of the young adults could not correctly add amounts of money in a problem similar to many business arithmetic situations. Both groups are entitled to subjective evaluation of performance levels on individual items. However, implicit in praise or criticism of the results is comparison to previous levels of mathematical achievement that is in no way indicated by National Assessment. The 1972-73 assessment will provide baseline data on the attainment of varied mathematical knowledge and skills; but not until the next assessment, when approximately half the exercises will be repeated, will NAEP indicate growth or decline of these attainments.

In examining results of individual exercises it is also very tempting to generalize strong or weak performance to broad classes of similar items. For instance, based on performance on the following exercise and a similar unreleased exercise, NAEP reported "Less than one half the 17 year olds and adults could determine the most economical size of the product." [14]

Problem: A housewife will pay the lowest price per ounce for rice if she buys it at the store which offers

	13	17	Adult
o 12 ounces for 40 cents	13%	10%	4%
o 14 ounces for 45 cents	9%	8%	5%
• 1 pound, 12 ounces for 85 cents	25%	34%	39%
o 2 pounds for 99 cents	46%	46%	47%
o I don't know	6%	3%	4%

The problem of drawing broad conclusions from sparse data is complicated by the fact that the roughly 200 exercises administered at each age level (not all given to each subject) are distributed among the 75 cells of a behavior/content exercise specification matrix. To help educators make use of the assessment results, NAEP has prepared two reports of selected results, Math Fundamentals and Consumer Math,

which collect items related to the specified themes. Furthermore, a special NCTM committee has prepared detailed analysis and interpretation of the results for presentation at the meetings and in the journals of NCTM.

NACOME has examined carefully the NAEP assessment scheme, the released items, and the various technical reports and journal interpretations that have been prepared.* While there is little virtue in repeating the details of those reports, there are several broad topic areas in which the assessment includes enough exercises to indicate reliable performance trends with important implications for school mathematics.

a) Arithmetic Computation. Though we have argued in earlier sections of this report that the importance of computational skill is diminishing in the modern world, the prominence of calculational proficiency in current school goals makes it a topic of interest. The assessment included 20 exercises on addition, subtraction, multiplication, and division of whole numbers -- most administered to all four age groups. The four sample exercises below illustrate a pattern which we judge to be strong performance. [15]

- Add:
$$\begin{array}{r} 38 \\ +\ 19 \end{array}$$

	9	13	17	Adult
Correct Response:	79%	94%	97%	97%

- Do the following subtraction:
$$\begin{array}{r} 1{,}054 \\ -\ 865 \end{array}$$

	9	13	17	Adult
Correct Response:	27%	80%	89%	90%

- Multiply:
$$\begin{array}{r} 38 \\ \times\ 9 \end{array}$$

	9	13	17	Adult
Correct Response:	25%	83%	88%	81%

*We are in debt to the NAEP staff and the NCTM interpretation committee for early access to their reports.

- Divide: $5\overline{)125}$

	9	13	17	Adult
Correct Response:	15%	89%	93%	93%

Unfortunately, very few computation exercises involved common fractions or integers, so NAEP suggests little of value in these important topic areas. The assessment did include 7 exercises on percent, many in problem solving situations, and performance was disappointing. The following sample exercise indicates the types of errors and level of performance. [16]

Problem: Candidate A received 70 percent of the votes cast in an election. If 4,200 votes are cast in the election, how many votes did he receive?

	13	17	Adult
Responses:			
Correct answer	10%	41%	62%
Correct process, wrong answer	2%	3%	5%
Attempt to divide (4200 ÷ 70)	29%	17%	6%
Attempt to add (4200 + 70) or subtract (4200 - 70)	16%	2%	-
Other unacceptable	20%	19%	15%
"I don't know"or no response	23%	18%	12%

b) Geometry and Measurement. Though geometric topics have long been standard fare in high school mathematics, recent innovations have attempted to introduce geometry in elementary and junior high school. Many of the NAEP geometry exercises involved only recognition or recall of names for various plane and solid figures. At all age levels the solid figures were much less readily identified. For instance, [17]

	9	13	17	Adult
Recall name:				
triangle	88%	-	-	-
sphere	2%	21%	46%	41%
cube	4%	26%	43%	54%
Identify the shape:				
rectangle	74%	-	-	-
cylinder	37%	58%	70%	68%
cube	39%	63%	74%	73%

When it came to applying geometric relationships, performance of 9 year olds was relatively poor and 13 year olds much better, though far from perfect. For instance, only 5% of the 9 year olds and 36% of the 13 year olds could calculate the diameter of a circle given the radius. Only 36% of the 9 year olds and 60% of the 13 year olds could calculate the distance between centers of two adjacent squares of the same size.

Another indication of school geometry emphasis and effectiveness comes from the NAEP measurement exercises. It appears that at all age levels students do not understand basic concepts of length, area, and volume. For instance, while 82% of the 9 year olds could accurately measure a 7 inch segment, only 48% could measure a 15 inch segment -- longer than the foot ruler they were given. Only 7% of the 13 year olds could calculate the area of square with perimeter 12 inches. Older respondents also had difficulty with area and volume problems calling for thoughtful application of concepts and formulas. [18]

Overall, the NAEP results suggest very modest progress toward enriched geometry programs K-8. Even at higher grade levels there appears to be need for greater emphasis on understanding basic concepts. Many respondents seemed too willing to apply any simple formula they remembered rather than to analyze the problem setting.

c) Problem Solving and Applications. The assessment included a wide variety of exercises that called for application of arithmetic or geometric skills and concepts to verbal problems. Learning to solve word problems is notoriously difficult, yet the assessment data indicate that when respondents understand the pertinent mathematical concepts, they can use the concepts in correct analysis of simple word problems. When they are not completely familiar with the underlying mathematics or when the problem is more complex, errors in problem analysis are more frequent. For instance, 75-80% of 13 year olds could subtract accurately when regrouping was needed, and 72% of this age group could solve the following problem: [19]

> A rocket was directed at a target 525 miles south of its launching point. It landed 624 miles south of the launching point. By how many miles did it miss its target?

On the other hand, though 17 year olds and young adults were reasonably proficient at basic arithmetic, the unit pricing problem cited earlier -- involving divisions, comparisons, and conversions of units -- caused considerable trouble.

Results from the first National Assessment do not provide a comprehensive or definitive picture of school mathematics achievement. They suggest topics in the curriculum that we judge worthy of increased emphasis and instructional research -- many of them perennial difficulties. The results on computation do not confirm charges that basic skills seriously deteriorated during the "new math" era. On the other hand, they suggest the need for greater attention to understanding basic concepts in topics like measurement and problem analysis.

Summary. Taken together, the achievement data that NACOME has examined suggest two broad conclusions about the state of school mathematics achievement.

1) When it has been possible to compare similar classes using traditional and modern mathematics texts, there has been a tendency for the traditional classes to perform better on computation while the modern classes perform better in comprehension.

2) There appears to have been a decline in basic scholastic skills since 1960. Mathematics achievement has shared in this decline. Recalling our observation in Chapter 1, that the reform movement in mathematics was not widely implemented in the classroom, there is still some evidence that secondary classes using a modern program have tended to resist the general decline in achievement.

In general, data that would supply a definitive picture of mathematics achievement in grades K-12 do not exist. The national picture is far more varied and complex than either proponents or critics of recent curricular innovation suggest in their current public debates.

5.2 Evaluation of Mathematical Abilities

The content and instructional styles of school mathematics are shaped on one hand by the educational desires and abilities of students and on the other by the expectations and resources of schools, expressed in their programs. In any school or class the individual students have a wide range of goals: to acquire specific mathematical skills, to prepare for specific employment opportunities, to develop broad intellectual skills, to attain prerequisites for further study, etc. These students bring to pursuit of their goals unique combinations of aptitude, interest, and previous learning. Evaluation is essential to determine student progress toward the goals and to help design optimal instructional programs. The schools have a wide variety of program goals: to raise overall student performance in areas of mathematical skill or understanding, to develop new curricular material, to improve effectiveness of teachers through in-service education, etc. Evaluation is essential to measure progress toward these goals and to plan program improvement. Thus there are many different purposes for evaluation in school mathematics and, accordingly, many conceivable techniques for accomplishing the various purposes. Unfortunately, evaluation in American mathematics education is is characterized by use of limited techniques inappropriately matched to goal assessment tasks.

The various techniques of evaluation fall into three broad categories, each with particular advantages and important limitations:

Evaluation by subjective methods of measuring student or program progress toward goals includes: teacher judgment of a student's achievement or attitudes; expert opinion on effectiveness of a curriculum; considered judgment of an evaluation team; or responses by program participants to an evaluation questionnaire. The advantage in this style of individual or program evaluation is that the personal evaluators can be chosen because of their close familiarity with the subject to be evaluated. They may be able to offer particularly insightful criticism and view the individual or program activities from a broad perspective. The major disadvantage is that the biases and prejudices of the evaluators are likely to have a strong effect on the results of the evaluation. In fact, it is almost impossible to find people who are sufficiently well informed to be good evaluators but have not already formed strong opinions of the person or program they are asked to evaluate.

For many individual students and educational programs, specific learning experiences are viewed primarily as means to achieving broader goals at a later time. Some students learn business mathematics to prepare for employment in sales or bookkeeping; some study analytic geometry as preparation for calculus. Many teacher in-service programs are designed to "sell" specific curricular innovations. In these situations delayed outcome data are natural evaluative measures. Success of students in subsequent employment or coursework and the innovative activity of participants in teacher education programs are true measures of effectiveness for their educational experiences. Unfortunately, this type of evaluation usually comes too late to be of any real use. For example, although student performance in college mathematics may be an excellent indication of accomplishment in secondary school, what is really needed is information on that student's likelihood of success prior to entering

college study. Similarly, if evaluation of an innovative curricular or instructional program must wait for such delayed outcome measures, an ineffective program may be supported long past the time it should have been discontinued.

Evaluation by testing consists of setting individual or group tasks which are related to goals in such a way that achievement of the goals would be reflected in better performance on the given tasks. Testing provides an immediate and objective approach to the evaluation of either individual student progress or the effectiveness of an educational program. On the other hand, a testing procedure can only sample the possible outcome of an individual student's experience or a program's activities. Most of the misuses and abuses of testing are associated with this sampling problem.

The objectivity and immediate feedback potential of testing make it such a desirable approach to evaluation that it is important to identify and eliminate the abuses and misuses of tests and test data. In this section we shall examine common testing materials and practices from this point of view.

Analysis of Testing Practices in Mathematics. There are at least five types of tests used in the schools today.

1. Teacher-constructed classroom tests
2. Text publisher's end-of-chapter or unit tests
3. Tests prepared by department, school, district, or state
4. Norm-referenced tests
5. Criterion-referenced tests

Teacher-constructed classroom tests and publisher's tests are largely used for grading and occasionally for diagnostic purposes. The scores provide the teacher with information concerning the progress each pupil is making in learning a particular topic. Strengths and weaknesses of individual students can be identified, enabling the teacher to give special attention if required. The performance of the class

on a classroom text also gives the teacher some indication of his or her success or lack of success in treating the particular topics. Teacher-constructed tests will continue to be an important source of diagnostic information on student performances. However, they cannot provide a rating of student performance on an absolute scale.

Tests prepared by a department, school, district, or state usually have the objective of evaluating particular programs. Although the scores may be used for the evaluation of individual student performance on a broader base, their main use is the comparison of groups of students participating in alternative or competing programs.

A norm-referenced test is a test carefully constructed to represent the content area common to many of the curricula used in the schools and normed with respect to an appropriate stratified sample of the national population at the given educational level. The results are reported either as normalized scores, percentile ranks, or grade level scores.

The grade level score is based on the average performance of students at each grade level for the given time of the year. Thus a grade level score of 3.6 represents the average performance of students six months into the third grade. Though grade level score reporting has a certain popular appeal, it has a number of serious shortcomings. First, it does not have a natural statistical interpretation in terms of the population of students at the grade level of interest. A student's grade level score is not determined by and does not explicitly identify his position in the distribution of scores for his age group peers. Second, in the popular mind, all students are expected to perform at grade level or above. However, from the norming process approximately half of the population will of necessity perform below grade level. Third, it is naturally expected that there will, on the average, be a unit increase in grade level score after a year's instruction. For students with scores

well below or above their grade level this is not reasonable. Finally, the grade level scores are easily misinterpreted. A bright third grader who receives a score of 5.5 is not necessarily performing like a mid-year fifth grader. As a third grader he may simply be doing very well on the problems appropriate for his grade level but might be able to do very few of the fifth grade level problems.

In view of the pitfalls of grade level scores it is unfortunate that test results are so frequently reported in this form. NACOME recommends that grade level score reporting be abandoned. A viable alternative for score reporting is the use of stanines which give the score as one of the integers 1 through 9. Each stanine score corresponds to a specific portion of the normalized score range and thus has a statistically valid interpretation. Stanine scores are not subject to the fine distinctions which are frequently but improperly made on the basis of normalized or percentile scores.

Norm-referenced tests are widely used both for individual student assessment and for program evaluation. As an instrument for individual student evaluation, the test provides a score (or possibly two or three subscores) which compare his performance with that of the norming population. Furthermore, the score (or scores) represent average performance in a wide range of content areas at several cognitive levels. Hence, areas of relative strength and weakness cannot be determined. This fact makes the test practically useless for diagnostic purposes and thus a poor instrument for evaluating student progress toward individual educational goals. In fact, the test is appropriate only in those few cases where the student's objective is directly related to overall school performance. The use of the College Board SAT test as a criterion for admission to college is a relevant example.

The standard norm-referenced tests are even less appropriate for program evaluation. Only rarely are programs directed solely

at an overall improvement in student performance. Usually the program has a specific set of objectives and the testing should focus on these objectives. It may be that the general impact of the program is of some interest, but this is of low priority compared to the impact in the specific areas represented by the objectives. The norm-referenced test scores which measure overall performance simply cannot provide the specific information needed to evaluate the impact in the areas of interest. Indeed, a specific program impact may be missed altogether since its effect can be masked by the effect of the program in other areas.

In many cases where norm-referenced tests are used for program evaluation the entire group of students participating in the program is tested. When the group is large, and this is the usual case, this results in unjustified over-testing. In program evaluation it is not the performance of the individual student but the performance of the group as a whole which is relevant. Testing an appropriate sample of students will provide reliable data on the performance of the group as a whole. This points up the inefficiency of administering a single many-item test for the purpose of program evaluation. Since the testing of a selected sample of students will provide sufficiently accurate data, a number of short test scales directed at specific program objectives may be administered to appropriately selected samples of students. In this way significant information concerning the effect of the program in a number of different areas of interest can be obtained while maintaining a very light testing load for each student.

In recent years there has been increasing use of another type of standardized test, the criterion-referenced test. It is normally concerned with a specific content area and the standardization consists in setting a percentage of correct responses which constitute mastery of the content area. Accordingly, these tests are sometimes referred to as mastery tests. Criterion-referenced tests are

frequently used as end-of-unit mastery tests in a programmed course of individualized instruction. However, the principle is applicable to many other testing situations. Since criterion-references tests concentrate on specific content areas, they can clearly be used as a diagnostic tool for individual student evaluation. Of course, the content areas must be selected so as to be relevant to the student's educational objectives. The level of mastery required may vary with the objective so that a standard mastery level may not always be appropriate. Nevertheless, the possibility of directly relating the test to the objective gives this type of test a distinct advantage in individual student evaluation. A danger associated with criterion-referenced testing is the possibility that the standards set in the testing program will become ceilings for student performance. Students should be encouraged to exceed the standard limits.

In principle, criterion-referenced tests may be used for program evaluation provided the tests are relevant to the objectives of the program. However, the mastery criterion may not be the most effective way of collecting the testing data. Clearly such a use of criterion-referenced tests involves a great deal of testing, though the burden on students can be alleviated by item sampling.

For purposes of program evaluation, the principle of using tests directed at specific objectives can be combined with the norm-referencing technique. Thus a collection of short scales directed at specific content areas can be normed on a suitable population. Using item sampling, if necessary, the scales are administered to the group of students participating in the program. In this way, information concerning the performance in each of the objective areas is obtained and can be compared to the corresponding performance of the norm group. The SCIMA program in California described in the next section is an example of such an evaluation scheme.

Since standardized, norm-referenced tests do not meet the general requirements of objective-directed testing, it is unfortunate

that they are so widely used for student and program evaluation. However, they will undoubtedly continue to be extensively used until suitable collections of carefully constructed, objective-directed test scales are widely available. The development of such collections as alternatives to the standardized tests now in use should be a high priority consideration of the testing industry.

Measuring Affective Variables. The preceding discussion of evaluation techniques and the testing results reported in section 5.1 focused almost exclusively on the cognitive outcomes of mathematics instruction. Many psychological tests have been used in research aimed at uncovering important relationships between cognitive and affective variables. To date findings have been minimal -- in spite of the fact that there is almost universal agreement that affective variables play an essential role in the learning of mathematics. New ideas are badly needed for appropriate and sensitive measures of the affective component in mathematics education.

Cultural Bias in Testing. In recent years I.Q. tests have come under severe criticism on a number of counts. The most serious objection has been the allegation that they penalize the poor or the culturally different. Since mathematics testing and I.Q. testing share some common features, this criticism has also been directed toward mathematics testing. Thus it is important to examine the extent to which this criticism is valid and to formulate testing principles which will insure that the tests are fair to students from a variety of cultural backgrounds.

Almost all mathematics tests have a certain reading component. This will be particularly prominent in tests with a large proportion of word problems. However, except for those which are purely computational, most problems will contain some descriptive material. Clearly a student who has limited English speaking ability or difficulties in reading comprehension will be handicapped in taking a

mathematics test. The handicap will be particularly severe if this test is a speeded test in which a premium is placed on understanding the problems quickly and doing them efficiently. Traditionally, computational tests have been speeded, since one of the objectives being tested is the ability of the student to perform calculations quickly and accurately. On the other hand, it is not so clear that speed is of major importance in mathematics understanding. It is the completeness and depth of the understanding which is of primary interest. The ability to comprehend a problem quickly and to do it efficiently may be some indication of depth of understanding, but a much better indicator is the ability to solve difficult and complex mathematical problems. Less use of speeded instruments for testing mathematical understanding would be a big step in reducing the reading comprehension problem. At the elementary level, the reading problem can be further reduced by having the verbal portion of the problem read to the students in a language they understand. At all levels the problem can be alleviated at the item construction stage by reducing the verbal component to a minimum, choosing the simplest possible terminology and replacing or supplementing the verbal statements with appropriate diagrams whenever feasible. With careful attention to item construction and proper test administration, mathematics testing should be practically bias-free as far as language barriers and reading comprehension are concerned.

Cultural background will determine to a large extent the nature of the mathematical concepts which children bring with them when entering school. It is the responsibility of the mathematics curriculum to correct and refine the concepts so that they can be used effectively as a common language of mathematics in later years. Hence, as long as the test items contain only those concepts developed in the school curriculum, there cannot be a serious cultural bias in the conceptual content of the test. One class of test item, however, falls outside of this category -- items treating applications of mathematics. Since the applications may be drawn from a wide variety

of human activities, cultural differences may easily affect the understanding of the framework of the problems. In order to avoid unfair bias in such items, the areas of application must fall well within the cultural experience of all members of the population being tested.

A third source of possible cultural bias in mathematics testing is in the testing operation itself. Testing assumes that the students will be stimulated favorably by the competitive situation to perform at the highest level of their abilities. It is known that individual students do not always respond in this manner. A few may react to the competitive situation by intentionally performing poorly. Others may respond to the pressure by becoming so tense that they perform badly in spite of themselves. Unfortunately, such behavior patterns may be culturally related. A history of poor performance on reading or I.Q. tests may lead to a negative attitude toward tests in general on the part of a cultural group. Under the circumstances, the testing operation cannot provide a valid measure of performance of this particular group. There are several ways in which a cultural bias of this kind can be overcome. First, many of the formal arrangements and activities associated with testing could well be eliminated (special seating, proctors, special timing, etc.). Second, testing should be incorporated as far as possible into the regular classroom activities. Third, the testing should consist of short scales, administered over several days if necessary, so that it does not become a big operation on a particular day. Finally, the test items should be interesting, so that a student's natural curiosity will stimulate him or her to a high level of performance.

5.3 State Assessment Schemes

In Chapter 2 we noted the recent emergence of state level objectives for mathematics instruction in grades K-12. In most states it appears that the objectives have been developed primarily to guide accountability-motivated mathematics assessment programs. The

assessment testing programs have typically been established after broad consultation with mathematics teachers, teacher educators, mathematicians, and interested laymen, and the most commonly stated purpose of testing is to provide background information for long-range educational planning. Yet informal reports to NACOME indicate growing concern among teachers about the competitive impact of comparing results from different schools and districts within a single state. Furthermore, there are unmistakable signs that the testing programs are beginning to influence the curriculum and instructional priorities in many states. Thus the substance, technical design, and use of state assessment have become a central concern for mathematics teachers across the country.

To serve as a basis for critical analysis and recommendations on future directions for mathematics assessment, NACOME collected descriptions of assessment schemes from each state. The specific content, procedures, and purposes of testing are extremely diverse and changing from year to year, but the following summary indicates the range of current practice. [20]

Twenty-two states now have regular programs of mathematics testing and ten more have conducted at least one recent statewide assessment. Among the states with regular testing programs, ten use norm-referenced tests, ten use criterion referenced tests, and two use a combination of both. Most testing is in grades K-9, but several states follow the National Assessment model of ages 9, 13, and 17 and two states focus on a twelfth grade "school leaving" examination of basic skills.

Uses of Standardized Tests in State Assessment. When the first state accountability and assessment programs were formulated, the predominant conception of testing was measurement against a standard of national, state, or local norms. In many states the basic assessment instrument is still a standardized achievement battery; but

these norm-referenced tests are being used to serve a variety of information gathering pruposes. For example:

1. Several states have assessment programs, but no specific mathematics education objectives. These states commonly use one of the standard commercial test batteries for testing with the intention of providing general information about trends in educational achievement at various grade levels. In these states assessment reports commonly indicate average grade equivalent performance for the state and for each major educational district within the state on the total battery and on each main sub-test. These scores are also compared to previous year scores, with some compensation for differences in aptitude and socio-economic background of the districts. This type of testing and reporting is used in Virginia, Mississippi, and Rhode Island. In Rhode Island the issue of test score comparison between districts has become a delicate political issue.

2. Several states have formal schemes of objectives for mathematics, yet assess achievement by standardized achievement tests. In Maryland the state has 6 broad goals for mathematics education, comparable to the 1972-73 NAEP behavioral categories of recall, manipulation, understanding, problem solving, reasoning, and appreciation. Each local district and school develops its own performance objectives within the state framework, yet the legislature accountability report is based on state-wide administration of the ITBS at grades 3, 5, 7, and 9. The report indicated grade equivalent performance of each county and school in the state, adjusted for differences in aptitude and socio-economic factors.

3. In Kentucky the state mathematics objectives are simply specified levels of performance on the CTBS. The tests are administered at grades 4, 8, and 11 in each education district of the state and results are reported in two ways. First are the state and district average grade equivalent scores on computation, concepts, and applications. Second is an item analysis of the test indicating performance on various clusters of items that make up the major subtests. For instance,

   > Comprehension of Equations -- The expected criterion level for fourth grade pupils was established as 66 percent. The average percentage of correct items for fourth grade statewide sample was 64 percent.

   The expected performance level is based on item data from the national norming sample. The use of item analysis data from administration of a standardized test is one way to make a norm-referenced test serve criterion-referenced

purposes. In Georgia and New Mexico results from use of the CTBS at various elementary grade levels have been reported in this form too.

4. Delaware has adopted another variation of the mixed norm- and criterion-referenced testing procedure. After developing statewide terminal mathematics objectives for grades 1, 4, and 8, the State Department of Instruction contracted with the Educational Testing Service to construct an examination testing achievement of those objectives. Since a high percent of the items used were drawn from a pool for which national norms were available, the report of results makes several comparisons -- Delaware versus national norms and versus Delaware in previous years. Pennsylvania has used a similar strategy in constructing a test of basic skills with numbers.

Though nationally standardized tests are widely used in state and local assessment efforts, the various attempts to tease specific achievement information from item analyses are symptomatic of growing dissatisfaction with testing programs that reduce mathematical achievement to several numbers indicating overall performance.

Uses of Criterion-Referenced Tests in State Assessment. If assessment data are to be used constructively in program improvement they must provide detailed analysis of the concepts and skills possessed by students. Thus the current trend is toward development of tests that measure student mastery of carefully delineated mathematical objectives. The variety of existing criterion-referenced testing programs is a measure of the variety in scope, detail, and organizing schema for state mathematics objectives.

1. The most common organizing framework for objectives is a grade level by content strand matrix. For instance, in Wisconsin, panels of teachers and mathematicians developed moderately specific behavioral objectives for grades K-8 arranged in strands like sets and numbers, numeration systems, order, computation, size and shape, sets of points, symmetry, etc. Another panel constructed test items and arranged them according to priority. In subsequent administration of the assessment test to a sample of 3rd and 7th graders (2000 per item) statewide criteria of performance were set at 75% correct for top priority items, 50% for second priority items, and 25% for third priority items (none included in first testing). The data are reported

in detail -- each objective is stated, the corresponding test item given, and performance of the state sample reported along with an evaluation of the performance. Patterns in the individual item results are summarized in overall recommendations. [21]

For example:

Objective/Item	State Sample	Evaluation
m57 Add rational numbers expressed in "terminating" decimal form.	a. 4.3%	E: Acceptable
5.87 + 25.603 + 0.95 =	b. 9.5%	
	*c. 81.4%	
A. 30.873	d. 4.4%	
B. 25.953		
C. 31.823		
D. 31.853		

2. New Jersey has developed mathematics assessment tests for grades 4 and 12 using an approach similar to that in Wisconsin. For the New Jersey test no levels of priority were established and no fixed criteria for acceptable performance. Furthermore, the test results are reported according to community type (urban, suburban, rural) and geographic region in the state.

3. Michigan has developed a criterion-referenced testing program based on minimal mathematical competencies at grades 4 and 7. The Michigan objectives are much more specific than those used in Wisconsin, each objective is tested with 5 appropriate items, and 80% correct is established as a criterion for success. A summary report from the State Department of Education highlights objectives for which over 85% of the students reached criterion and others for which criterion was attained by fewer than 35% of the students. The detailed interpretive report emphasizes diagnostic and remediative analysis with tips for improved teaching going well beyond the simple acceptable/unacceptable judgment.

4. As part of the accountability interest in assuring that students have minimal mathematical skills for successful citizenship, several states have developed criterion-referenced tests for use in twelfth grade. Wisconsin, Missouri, Georgia, and New Mexico are among the states that have recently initiated this type of "school leaving" exam.

5. In Georgia and Florida, teams of mathematicians and teachers have formulated mathematics objectives for various grade levels. Then the state departments of education contracted with commercial testing companies to develop criterion-referenced tests based on those objectives. In both states the objectives are organized into content strands; however,

the Florida objectives are viewed as "priority objectives", not the outline of an entire mathematics program.

The mathematics assessment programs of many states are clearly influenced by the conceptions of mathematical achievement and strategies for measurement developed in NLSMA and National Assessment. In such states the mathematics objectives and test items are organized in content/behavior matrices. As is characteristic of such schema guided assessments, the emphasis is not on a single test score, but on patterns of achievement in many different facets of mathematics.

6. After many years of statewide testing with nationally standardized tests, California has recently converted to the objectives referenced SCIMA -- State of California Inventory of Mathematical Achievement. The SCIMA testing matrix has five content strands (arithmetic, algebra, geometry, measurement, probability and statistics) and three behavior levels (computation and fact knowledge, comprehension, application). Tests are then designed and reported to indicate performance in each of the resulting 15 skill areas and many sub-areas. As is common in such matrix testing programs, the various test items are administered to only a sample of California students at each grade level -- permitting inclusion of many more items in the test without putting an imposing burden on each student.
7. In Minnesota, Maine, Illinois, and Wyoming the National Assessment model of achievement testing has been the basis of state assessment programs. Though modified to meet local needs, this generally implies a content/behavior matrix for organizing the test items, testing at age intervals similar to those of NAEP, and use of selected NAEP released items within the state tests. These testing schemes will lead to criterion-referenced reporting, with an opportunity to compare state performance to national results on many items.

These descriptions suggest the range of existing or emerging state mathematics assessment procedures. NACOME had no way of determining a reliable profile of testing purposes and procedures at the local school district level. Testing serves more diverse purposes at the local level -- thorough measurement of the abilities possessed by each student along with more general assessment of

program effects. Furthermore, the practical impact of test content, procedure, and reporting is likely to be greater at the local school level.

In the rush to implement accountability related testing programs at all school levels, the wide ranging implications for school curricular and instructional practice have frequently been all too cursorily considered. The variety of existing assessment programs might be taken as evidence that states and school districts are acquiring test data uniquely suited to their educational objectives and informational needs; on the other hand, many schemes seem based on narrow views of the goals of mathematics instruction and produce information likely to be of little constructive value for improvement.

5.4 Summary

At the beginning of this chapter we posed three questions on the extent to which school programs are developing those mathematical skills and attitudes essential for effective participation in our contemporary technological world. The data present an incomplete pattern of mixed success and failure that must be interpreted hesitantly. Part of this difficulty of interpretation can be attributed to lack of information on existing curricular and instructional practice. But the NACOME analysis suggests fundamental problems in current practices of evaluating mathematics education.

Most of the methods currently used for evaluating and reporting program effectiveness are not sensitive to the specific objectives of the programs and are inefficient in terms of the time and effort required. Many testing programs use instruments that provide such crude measures of achievement that they have limited value for improving instructional programs or assessing an individual student's educational needs.

Evaluation is an essential and useful phase of any instructional program. But to make evaluation play a positive and effective role in school mathematics today there is an urgent need to develop a much broader collection of measurement techniques and instruments and to match these evaluation tools more appropriately to the varied purposes of evaluation.

CHAPTER 6. RECOMMENDATIONS AND PERSPECTIVES

This survey covers a vast and jumbled mathematical landscape. It is more than the proverbial jungle; there are obvious mountain peaks and gaping chasms. As a survey it came in a period of confused and changing boundaries amidst the partially receding waters of the "new math" deluge. It attempts to chart what is because it is important to do so now, knowing full well that the outcome may resemble those amusing early maps of the New World. It would be presumptuous for this small group, awed by the survey task itself, to formulate a comprehensive set of recommendations on the future course of mathematics in the schools. Nevertheless, having charted the waters, however roughly, we have come face to face with some unmistakable topographical features and feel that it is valid to call these to the attention of the mathematical community.

A. Policy Recommendations

Recommendation 1: Anti-Dichotomy

In the creation, introduction, and support of mathematics programs, neither teachers, educational administrators, parents, nor the general public should allow themselves to be manipulated into false choices between

the old and the new in mathematics
skills and concepts
the concrete and the abstract
intuition and formalism
structure and problem-solving
induction and deduction

The core of every mathematics program should contain a judicious combination of both elements of each pair with the balance, proportion, and emphasis between the two being determined by the goals of the program and by the nature, capabilities and circumstances of the students and teachers in the program.

Furthermore, little is communicated by polarization of positions about terms and slogans that have long since lost agreed-upon meanings. Therefore, we recommend that the term "new math" be limited in its use to describe the multitude of mathematics education concerns and developments of the period 1955-1975 and that reference to current school mathematics, its status, its trends, and its problems be made only in such common-noun terms as the "present mathematics program", "current school mathematics", "contemporary mathematics teaching", etc.

Recommendation 2: Quality Education

Those who are concerned about the education of young people must reaffirm their commitment to full, comprehensive mathematics education for all youngsters regardless of race, national origin, or sex, and to the encouragement of each to pursue mathematics so as to make maximum use of his or her mathematical talents.

Implications:

a) that every child is entitled to the mathematical competencies necessary for daily living in today's civilization, but the concept of "basic skills" essential to the consumer and the citizen be defined to include more than computational skill -- also abilities to deal intelligently with statistical information, to reason logically and think critically.

b) that minimum skills cannot be allowed to become ceilings of performance for any youngster.

c) that the provision and support of qualified teachers is primary to the accomplishment of these goals.

d) that teachers at all levels and in all geographical areas should have the opportunity to select from among the growing array of alternative teaching styles

and materials those that best meet the needs of their students.

e) that teachers of mathematics continue to be supported within their school systems by qualified resource specialists in mathematical curriculum and instruction.

Recommendation 3: Curriculum Content

Curriculum content, subject to the flux of accelerating change in all areas of our society, cannot be viewed as a fixed set of goals or ideas; it must be allowed to emerge, ever changing, responsive to the human and technological lessons of the past, concerns of the present, and hopes for the future. With this in mind, no definitive curriculum can ever be recommended. At benchmark 1975 the National Advisory Committee on Mathematics Education sees the following recommendations as reasonable and essential features of a contemporary mathematics curriculum.

a) that logical structure be maintained as a framework for the study of mathematics.

b) that concrete experiences be an integral part of the acquisition of abstract ideas.

c) that the opportunity be provided for students to apply mathematics in as wide a realm as possible -- in the social and natural sciences, in consumer and career related areas, as well as in any real life problems that can be subjected to mathematical analysis.

d) that familiarity with symbols, their uses, their formalities, their limitations be developed and fostered in an appropriately proportioned manner.

e) that beginning no later than the end of the eighth grade, a calculator should be available for each mathematics student during each mathematics class. Each student should be permitted to use the calculator during all of his or her mathematical work including tests.

f) that the recommendations of the Conference Board of the Mathematical Sciences 1972 committee regarding computers in secondary school curricula be implemented.

NACOME especially underlines recommendations:

-- that all students, not only able students, be afforded the opportunity to participate in computer science courses,

-- that school use of computers be exploited beyond the role of computer assisted instruction or computer management systems,

-- that "computer literacy" courses involve student "hands-on" experiences using computers.

g) that all school systems give serious attention to implementation of the metric system in measurement instruction and that they re-examine the current instruction sequences in fractions and decimals to fit the new priorities.

h) that instructional units dealing with statistical ideas be fitted throughout the elementary and secondary school curriculum.

Recommendation 4: Teacher Education

Colleges of education, professional mathematics education organizations, accrediting agencies of teacher certification, and the mathematics community must cooperate to produce mathematics teachers knowledgeable in mathematics, aware of, oriented to, and practiced in a multitude of teaching styles and materials and philosophically prepared to make decisions about the best means to facilitate the contemporary, comprehensive mathematics education of their students. Further, the above bodies, together with local school boards and organizations representative of teachers must continually facilitate the maintenance of teachers' awaremess of and input to current programs and issues.

Implications for pre- and in-service teacher training:

a) that professional organizations continue to update and publish the profession's view of the educational needs of mathematics teachers and that professional organizations take an active and aggressive role in apprising decision-makers in teacher education, certification, and accreditation of these views.

b) that a joint commission of NCTM and MAA be established to present a united position on requirements for the education of pre-college mathematics teachers.

c) that the professional organizations in mathematics education take initiatives to insure that mathematics educators have a role in decisions relating to the preparation of specialist teachers in special education, early childhood education, bilingual education, career education and other areas in which mathematics is part of the curriculum.

d) that mathematics specialists with broad- and long-range perspective concerning the nature of mathematics and its role in society (mathematicians and mathematics educators) maintain a prominent role in decisions concerning the mathematical competencies of teachers, both in design of teacher education programs and in the certification of teachers.

e) that neither teacher education nor certification procedures be based solely on competency- or performance-based criteria without a sound empirical rationale.

f) that the background of instructors in both pre-service and in-service courses for teachers include not only the relevant mathematical competence but both current experience and interest in the mathematical curriculum of the level those teachers will teach.

g) that, since the successful implementation of any thrust in school mathematics depends on the realistic acceptance of that thrust by teachers, programs seeking national acceptance must identify the factors promoting such acceptance and integrate these into in-service workshops. Among the factors considered should be:

   -- the conditions under which the teacher is attending the institute,

   -- the teacher's opportunity to make input into the program,

   -- the teacher's opportunity to adapt methods or materials to his or her own style of classroom instruction,

   -- the opportunity to air misgivings and apprehensions and to brainstorm both future difficulties of implementation and alternatives for avoiding or handling them.

h) that school districts, teacher organizations, and sponsoring agencies of teacher education programs should work together to identify the conditions that will promote teacher participation in in-service programs. Possible factors might be released time, educational leave time, university credit, stipends,

credits toward other negotiated benefits, and incentive teaching materials.

i) that teacher education place emphasis in the following areas:

   1) the development of process abilities, that is, abilities in logical reasoning and problem solving, and methods of developing these abilities in children,

   2) development of teacher judgmental abilities to make intelligent decisions about curricular issues in the face of growing outside pressure for fads and uninformed policy,

   3) recognition that skills of statistical inference and the ability to deal intelligently with collections of information are among the essential, minimal skills required by every person in today's world,

   4) appreciation of the uses and applications of mathematics in the solution of "real world" problems,

   5) development of skills in teaching the effective use of computing and calculating machines in solving problems,

   6) for secondary teachers, literacy in at least one problem solving programming computer language and grasp of the issues in computer literacy,

   7) preparation of new teachers to enter realistically the existing school systems as well as to participate in emerging trends.

Recommendation 5: Affective Domain

Despite legitimately changing national priorities, where humanities or sociological concerns supersede the technological, mathematics remains an ever-growing requisite for both personal living and the maintenance of the industrial and scientific society we have become. Positive attitudes and expressions on the part of parents, teachers, and the general public with regard to mathematics that leave the student free to encounter without prejudice the realities of mathematics, with its own attractions and difficulties, are fundamental in successfully providing the young with a mathematical education that will fulfill their personal needs as citizens and provide

the pool of mathematical talent requisite for the social needs of the nation.

Implications:

a) that the affective as well as cognitive domains in mathematics should be the subject of constant and programmatic attention,

b) that basic research into the affective domain specifically vis-a-vis mathematics should be pressed,

c) that attempts to create more appropriate and sensitive instruments for assessing the affective domain should be mounted,

d) that the items or scales referred to in c) above then be included in national and regional assessments and in all major program evaluations,

e) that pre-service and in-service programs, both general and those related to specific projects should formally embody objectives in the affective domain and program elements to achieve them,

f) that the unfounded assumption that mathematics is more a subject for males than a subject for females is to be vigorously opposed.

Recommendation 6: Evaluation

Given the prominence and value of evaluation in the scheme of education today, more critical attention needs to be given to this area.

Implications:

a) that evaluation instruments be selected after program or individual goals are identified and that they be matched to these goals,

b) that grade-level score reporting of student performance on standardized tests be abandoned,

c) that an intensive effort be made to develop objective-directed tests to replace the standardized norm-referenced tests now commonly used for student and program evaluation.

d) that sampling techniques should be used in program evaluation wherever appropriate, to minimize the over-testing problem,

e) that evaluation results be reported in a multi-component form corresponding to the multiplicity of goals normally associated with education programs,

f) that extreme care be used in test construction and administration to minimize potential cultural biases,

g) that evaluators should be more sensitive to the effects upon performance of certain factors of testing conditions, e.g. time limitation, over-testing, lack of motivation, unfavorable physical conditions, attitudes of test administrators and teachers.

B. Recommendations for Research and Development

Recommendation 1: Needed Research

While continuing research is essential to every empirical issue in mathematical education, there are particular issues either of such overriding importance or for which the evidential base is so weak that we believe attention should be called to them. The following is by no means an inclusive list but it represents areas that appeared notable in the course of our survey as representing areas of special need.

a) Research on objective means for identifying good teaching and the characteristics of the effective teacher is needed. Present methods for identifying effective teaching models are largely subjective and impressionistic. Yet fundamental current issues in teacher education and teacher certification, such as the definition, recognition and development of teaching "competencies", cannot be satisfactorily resolved without a sound research base relating teacher characteristics and behaviors to successful educational outcomes and accomplishment of goals.

b) There should be continuing attempts to find a sound empirical basis for the recommendation of particular patterns, methods, and materials of instruction and of particular instructional and curricular organization. Needed are extensive evaluations of programs and comparative studies of alternative programs.

c) Considerable study is called for concerning the complex interrelationships among variations in teaching and learning styles and among variations in such styles and instructional staffing patterns and among these variables and content goals.

d) Once goals are clearly established concerning desired computational ability, research is needed to identify the techniques and balance of rationalization and practice, that are optimal for attainment of these goals.

e) Continuing research is needed in the affective domain, on variables associated with the development of attitudes and motivation and the relationship of these variables to achievement outcomes.

f) Little is known about the effective, optimal balance or interaction of informal and rigorous modes of expression in learning mathematics and its relationship to content, student ability, and experience.

g) Research is urgently needed concerning the uses of computing and calculating instruments in curriculum at all levels and their relationship to a broad array of instructional objectives.

h) Evaluation of the many alternative programs that are oriented toward the application of mathematics to varied problems or other content areas is needed. In particular, the success of specific instructional variables in achieving goals of modeling, application, problem formulation, development and selection of strategies, interpretation of information, discovery of relationships, etc. demands investigation.

Recommendation 2: Needed Information

Throughout the report, the committee has indicated areas where we have been hampered by the lack of information. Without listing these in detail we will note broad categories which seem to call for extensive information-gathering.

a) Extensive and detailed information about classroom practice is an urgent need. The status of mathematical education can certainly not be known until we have far more dependable data on what actually happens in the classroom. Any attempt to link cause to the assessed effects of testing programs is suspect until such information is available.

b) We know shockingly little about the preparation of teachers. Surveys are needed to determine common practices, program requirements, etc. in teacher education. In addition, it is suggested that teacher education institutions not only evaluate their programs in follow-up studies of their graduates but initiate means of sharing these findings on a broad scale.

c) A large-scale assessment of present needs for in-service education is timely; such assessment, to be useful, will depend upon the development of instruments more sophisticated than just polls of teacher opinion as to their perceived needs.

Recommendation 3: Needed Curriculum Development

The improvement and updating of curricular elements and material is a perpetual need, but there are particular areas considered in this report where new curricular organizations, instructional materials, and courses are of urgent concern.

a) Instructional materials at all levels in: the use of calculators, applications and modeling, statistics and the general ability to collect, organize, interpret, and understand quantitative information, combinatorial mathematics, and metric system measurement.

b) Curricular revision or reorganization in the light of the increasing significance of computers and calculators.

c) Curricular revision of the relevant components of the total program in light of the increasing use of the metric system in measurement.

d) Integration of statistical ideas throughout the curriculum at all levels. Some ways in which this might be done are:

   1) Use statistical topics to illustrate and motivate mathematics.

   2) Emphasize statistics as an interdisciplinary subject by encouraging the insertion of statistical ideas into the study of the natural, physical and social sciences and the humanities.

   3) Develop several separate courses dealing with statistics to meet the most varied local conditions. Possible courses are:

      a) a ninth grade statistics course available to all students, with no algebra prerequisite. This could probably be the most useful mathematics course for the non-college bound or any student who as consumer and citizen must cope throughout life with numerical information.

      b) a senior year statistics course with a probability prerequisite.

c) interdisciplinary courses oriented towards computers and statistics and social-, natural- and physical-science courses using statistical tools.

e) Recommendations of the CBMS 1972 Committee on Computers in the Secondary School are supported by NACOME.

1) preparation of a junior high school course in "computer literacy", general understanding of the capabilities and limitations of computers and their role in our society.

2) preparation of text materials for follow-up courses in computing, modules which integrate computing into high school mathematics courses, and other modules which utilize computers in simulating the behavior of physical or social phenomena.

3) development of special programs for high school students showing unusual aptitude and promise in computer science.

4) a major effort aimed at making vocational computer training more generally available and at the same time improving the quality of such training.

f) Techniques and materials which will support and effectively develop abilities in problem solving, in logical reasoning, and critical thinking.

g) New and imaginative approaches to the geometry in high school, junior high school and elementary school. In particular, a rethinking of the role geometry should play in the objectives and goals of the mathematics curriculum and its relationship to the rest of the mathematics program would be timely and valuable.

h) Effective materials and techniques for remedial instruction.

i) New and revised teacher education programs at both pre-service and in-service levels. The emphases recommended above all have implications for changes or new priorities in the teacher's education, of both mathematical and pedagogical nature.

j) Collections of short test scales directed at specific objectives as alternatives to the general purpose standardized test now widely used.

k) Appropriate and sensitive measures for the affective component in mathematics education.

Mathematics is more than just a school subject. It is a national resource, a national concern and, at times, a national issue.

School mathematics is in an unusual state today. Long enshrined as a unique and well-supported discipline with a clear-cut and almost monolithic identity, it is suddenly beset with many troubles -- an identity crisis brought on by the usual causes: internal confusion and loss of clear-cut direction and external changes in familiar support and status structures.

Current cultural preoccupations no longer award mathematical, scientific and technological disciplines the first place of honor (and funding) that has been the case for many years. The centrality of mathematics to national needs triggered by the Sputnik launching is no longer felt. Young people who formerly flocked to mathematics because it was so important and had such national status now look elsewhere. The popularity of mathematics and its funding by public and private agencies has greatly waned. At the same time, a plateau has been reached in the two-decade series of developments referred to as the "new math". Whatever its achievements (and they were many), it also has enough problems and unfulfilled goals to generate a host of critics among educators, parents and even politicians.

While some demythologizing of mathematics is probably justifiable, it may be that these two forces have too severely depreciated the role of mathematics (and its consequent support) in the eyes of many not only in the profession but, more regretably, among educators, government, and the general public. Yet whatever alternative fields of scholarship take precedence over mathematics and whatever external criticisms of developing mathematics programs arise, our world remains a scientific, technological, and industrial world and mathematics remains essential to its caretaking and improvement. It is necessary to reaffirm the continued importance of mathematics to society and to the individual.

It is also necessary to appraise criticisms realistically. For that, one must realize that all of American education has been deeply

conditioned by recent cultural changes. School-work attitudes, habits, and motivations have dramatically altered and, as a consequence, have definitely reduced many formerly expected educational outcomes. Many criticisms aimed at the "new math" are actually a response to this general educational decline, while other criticisms remain addressed directly to the mathematical program itself.

Furthermore, we as a society seem to be emerging from a naive period in which the simplistic assumption was held that education could provide the solution for most social ills. We are confronted with increasing evidence that the school is only one of a vast complex of forces which influence the achievement of children in scholastic areas. Thus a serious study of changes in educational outcomes must be put into the context of the dynamics of the society as a whole.

A positive side of the current wave of criticism is the evident interest and involvement in school mathematics of many important groups. This interest and even this criticism are welcomed, with the hope that they will be responsible and responsive to dialogue and interaction and will reflect the appropriate roles and competences of the critical bodies. Legislatures and governmental bodies have a legitimate interest in educational outcomes -- but seldom the professional depth of educational expertise. Their voice is important -- but should be raised with restraint and should remain in the policy sphere. Parents should remain interested and vocal -- but should make every effort to become informed before becoming critical. Teachers, individually and through their various professional agencies, should have an increasing voice in determining the parameters of mathematical programs in the schools.

All of these groups should recognize that mathematics is in no way a less appropriate subject for female than for male students and that national interest as well as personal justice indicates that

everything be done to open mathematical horizons and opportunities equally to both sexes. In all, mathematics education today is much like a healthy but not untroubled teen-ager. Its pre-teen period of enthusiastic preoccupation with the relatively clear-cut issues of content development has given way to a confused recognition of additional and much more elusive problems.

The school mathematics community is very much aware of and deeply involved with a host of problems: the diverse needs of many subcultures in the schools; questions of teaching and learning theory; the tremendous gap between theory and practice; problems of evaluating the merits of educational programs; the difficulties of acceptance and implementation even for programs of proven worth; pivotal issues of pre service and in-service teacher education so vital to this implementation. But the complexity of these problems compared to the earlier questions of content and curriculum is such that they have left the mathematical education community momentarily unsettled and unsure of direction and strategy. The teen-ager has acquired a healthy awareness of the broader issues of maturity.

Mathematics education today is the healthier because it has lost a certain simplistic view of its educational challenges and an undue certainty about its answers. It has embraced more fully problems that have always been its to address and is beginning an era of new endeavor -- charting, exploring and hoping to conquer more formidable reaches than ever before.

GLOSSARY OF ACRONYMS

ASA	American Statistical Association
CAI	Computer Assisted Instruction
CBTE	Competency Based Teacher Education
CEEB	College Entrance Examination Board
CMI	Computer Managed Instruction
COLAMDA	Committee on the Low Achiever in Mathematics, Denver Area
CSMP	Comprehensive School Mathematics Project
CTBS	Comprehensive Tests of Basic Skills
CUPM	Committee on the Undergraduate Program in Mathematics (a committee of MAA)
IPI-Math	Individually Prescribed Instruction-Mathematics
ITBS	Iowa Tests of Basic Skills
MAA	Mathematical Association of America
Madison Project	A curriculum and instructional development project located for many years at Syracuse University and Webster College
MINNEMAST	Minnesota Mathematics and Science Teaching Project
NAEP	National Assessment of Educational Progress
NCES	National Center for Education Statistics
NCTM	National Council of Teachers of Mathematics
NIE	National Institute of Education
NLSMA	National Longitudinal Study of Mathematical Abilities
NSF	National Science Foundation
PBTE	Performance Based Teacher Education

Project ONE	A curriculum development project located at the Educational Development Center in Newton, Massachusetts
RBS	Research for Better Schools, a regional educational laboratory in Philadelphia
SAT	Scholastic Aptitude Test
SCIMA	State of California Inventory of Mathematics Achievement
SMSG	School Mathematics Study Group
SSMCIS	Secondary School Mathematics Curriculum Improvement Study
UICSM	University of Illinois Committee on School Mathematics
UMMaP	University of Maryland Mathematics Project
USMES	Unified Science and Mathematics for Elementary Schools
WYMOLAMP	A project developing material for low achieving students, located in Wyoming

NOTES

Chapter 1

1. Commission on Mathematics. Program for College Preparatory Mathematics. New York: College Entrance Examination Board, 1959.

2. Bruner, Jerome. The Process of Education. New York: Vintage Books, 1960, p. 33.

3. Ibid., p. 18.

4. Williams, Irene. "A Progress Report on the Implementation of the Recommendations of the Commission on Mathematics." The Mathematics Teacher 63 (October 1970): 461-468.

5. Ostendorf, Logan. Summary of Course Offerings and and Enrollments in Public Secondary Schools, 1972-73. Washington, D.C.: U.S. Government Printing Office, 1975.

   See also: Wright, Grace S. Subject Offerings and Enrollments in Public Secondary Schools, 1960-61. Washington, D.C.: U.S. Government Printing Office, 1965.

6. This table is adapted from information given by reports cited in [5].

7. Norris, Eleanor L. and Bowes, John E. National Assessment of Educational Progress: Mathematics Objectives. Denver: Education Commission of the States, 1970.

   Additional information on items used in the assessment was provided by NAEP staff.

8. Results of this survey have not been formally published yet, but information was made available to NACOME by NCTM.

9. Kline, Morris. Why Johnny Can't Add. New York: St. Martin's Press, 1973.

10. Begle, E. G. "Review of Why Johnny Can't Add." The National Elementary Principal 53 (January/February 1974): 26-31.

11. Ibid.

Chapter 2

1. Hilton, Peter. "What is Mathematics?" in Michael Otte (ed.) Mathematiker über die Mathematik. Berlin: Springer-Verlag, 1974.

2. This table is developed from information in the reports cited in [5], Chapter 1.

3. Bukosky, William J. and Korotkin, Arthur L. Computing Activities in Secondary Education. Washington, D.C.: American Institutes for Research, 1975.

   See also Korotkin, Arthur L., Darby, Charles A., and Romashko, Tania. A Survey of Computing Activities in Secondary Schools. Washington, D.C.: American Institutes for Research, 1970.

4. CBMS Committee on Computer Education. Recommendations Regarding Computers in High School Education. Washington, D.C.: Conference Board of the Mathematical Sciences, 1972.

5. Kelley, John L. and Lansing, Ira G. "Some Implications of the Use of Hand Calculators in Mathematical Instruction." Bloomington: Indiana University Mathematics Education Development Center, 1975.

6. Ten Goals of Quality Education. Harrisburg, Pennsylvania: State Department of Education, 1973.

7. Minimal Performance Objectives for Mathematics. Lansing, Michigan: State Department of Education, 1973.

8. Guidelines to Mathematics K-6. Madison, Wisconsin: State Department of Public Instruction, 1970.

9. Priority Objectives for Communication Skills (Reading and Writing) and Mathematics in Florida Grade 3 (1973-74). Tallahassee, Florida: Department of Education, 1973.

10. Mathematics Framework for California Public Schools, Pre-Grade 12 (Proposed). Sacramento, California: State Department of Education, 1974.

11. Kentucky Educational Assessment Program, 1973. Frankfort, Kentucky: Department of Education, 1973.

Chapter 3

1. Dilworth, Robert P. and Warren, Leonard. Final Report: Specialized Teacher Project, 1971-72. Sacramento: California State Department of Education, 1973.

2. Conant, Eaton H. Teacher and Paraprofessional Work Productivity. Lexington, Massachusetts: D.C. Heath Company, 1973.

3. This is the survey described in detail later in the chapter, results not yet published in any journal.

4. Jamison, Dean, et. al. "The Effectiveness of Alternative Instructional Media: A Survey." Review of Educational Research 44 (Winter, 1974): 1-67.

5. Kelley, Jonathan and Kelley, John L. "Recommendations for Improving the Mathematics Programs in the Berkeley Schools." unpublished manuscript shared with NACOME.

6. Ibid.

7. See [2].

8. Olson, M. N. Identifying Predictors of Quality: An Examination of Eleven Internal Classroom Variables in Relation to a School System Criterion Measure. Doctoral Dissertation Columbia University, 1970.

9. See [5].

Chapter 4

1. Committee on the Undergraduate Program in Mathematics. Recommendations on Course Content for the Training of Teachers of Mathematics. Washington, D.C.: MAA, 1961.

2. Dubisch, Roy. "Teacher Education." In Mathematics Education. 69th Yearbook of the National Society for the Study of Education. Chicago: NSSE, 1970.

3. Ibid.

4. This is the exploratory survey described in Chapters 1 and 3.

5. Committee on the Undergraduate Program in Mathematics. Recommendations on Course Content for the Training of Teachers of Mathematics. Washington, D.C.: MAA, 1971.

6. Guidelines for the Preparation of Teachers of Mathematics. Reston, Virginia: NCTM, 1973.

7. MacIntosh, Jerry and Jackson, Roderick. Contemporary Practices in Secondary Mathematics Teacher Education. Bloomington, Indiana: Research Center for Language Sciences, 1973.

8. Guidelines for the Evaluation of Collegiate Mathematics Programs. Washington, D.C.: MAA, 1975.

9. Commission on Education of Teachers in Mathematics. Summary of a Survey of Mathematics Teachers. Reston, Virginia: NCTM, 1974.

10. Dilworth, Robert P. and Warren, Leonard. Final Report: Specialized Teacher Project, 1971-72. Sacramento: California State Department of Education, 1973.

11. Schmieder, Allen and Yarger, Sam J. Teaching Centers: Toward the State of the Scene. Washington, D.C.: American Association of Colleges for Teacher Education, 1975.

12. Carroll, Stephen J. and Ryder, Kenneth F. The Supply of Elementary and Secondary Teachers. Santa Monica: RAND Corporation, 1974, p. 103.

13. Ibid.

14. Atkin, J. Myron and Raths, James D. "Changing Patterns of Teacher Education in the United States." OECD, 1974.

15. Schmieder, Allen. Competency-Based Education: The State of the Scene. Washington, D.C.: AACTE, 1973.

16. Atkin, J. Myron. "The Rhetoric and Reality of Educational Change." Educational Technology 13 (November, 1973): 16-20.

17. Heath, Robert W. and Nielson, Mark A. "The Research Basis for Performance-Based Teacher Education." Review of Educational Research 44 (Fall, 1974): 463-484.

Chapter 5

1. Kenosian, Joyce. Statewide Report of Pupil Evaluation Program Test Results. Albany, New York: State Education Department, 1974.

2. Data supplied by the research director of the San Diego City Public Schools.

3. Law, Alex and Tardif, Robert. The Results of the 1971 and 1972 SCIMA Test Administrations. Sacramento: California State Department of Education, 1973.

4. Austin, Gilbert and Prevost, Fernand. "Longitudinal Evaluation of Mathematical Computational Abilities of New Hampshire's Eighth and Tenth Graders, 1963-1967." Journal for Research in Mathematics Education 3 (January, 1972): 59-64.

5. Reports from state testing in these states were supplied to NACOME by state supervisors of mathematics.

6. Dr. James Braswell of the Educational Testing Service provided extensive data on the performance of students taking various College Board tests.

7. Hieronymus, A. N. and Lindquist, E. F. Manual for Administrators, Supervisors, and Counselors: Iowa Tests of Basic Skills, Multi-Level Edition. Boston: Houghton-Mifflin, 1971.

8. Data supplied by the California Test Bureau.

9. Garigliano, Leonard. "Arithmetic Computation Scores: or Can Children in Modern Mathematics Programs Really Compute?" School Science and Mathematics (June, 1975): 399-412.

10. Beckmann, Milton W. "Eighth Grade Mathematical Competence Fifteen Years Ago and Now." unpublished manuscript shared with NACOME.

11. The discussion of NLSMA is based on drafts of an interpretive book prepared by NLSMA staff but not yet published.

12. Begle, E. G. "Review of Why Johnny Can't Add." The National Elementary Principal 53 (January/February, 1974): 26-31.

13. Math Fundamentals: Selected Results from the First National Assessment of Mathematics. Denver: National Assessment of Educational Progress, 1975.

14. Consumer Math: Selected Results from the First National Assessment of Mathematics. Denver: National Assessment of Educational Progress, 1975, p. 35.

15. See [13].

16. See [14]

17. Carpenter, Thomas P., et. al. "Results and Implications of the NAEP Mathematics Assessment: Elementary School." The Arithmetic Teacher 22 (October, 1975): 438-450.

18. Carpenter, Thomas P., et. al. "Results and Implications of the NAEP Mathematics Assessment: Secondary School." The Mathematics Teacher 68 (October, 1975): 453-470.

19. See [17].

20. Information for this discussion came from reports of state assessment procedures provided for NACOME by members of the Association of State Supervisors of Mathematics.

2 . Interpretive Report on Wisconsin State Mathematics Assessment, Spring 1974. Madison: State Department of Public Instruction, 1974: p. 44.